Stochastic Monitoring and Testing of Digital LTI Filters

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Introduction

Overall goal: "Functional verification" of digital systems

Application: Testing and monitoring of digital circuits
(e.g., manufacturing defects, design flaws)

Monitoring versus testing:

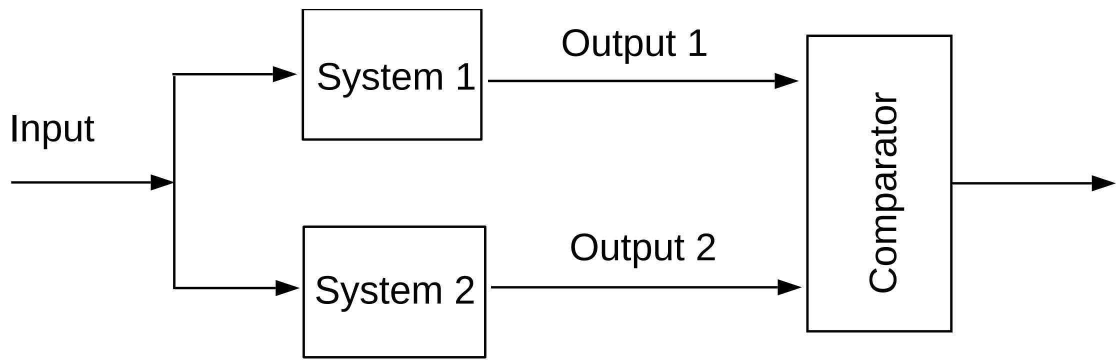
- *Testing:* "Off-line," can control inputs (e.g., testing sequence)
- *Concurrent testing:* "On-line," can access inputs (but not control)
- *Monitoring:* "On-line," cannot access inputs

Desirable features:

- High fault coverage
- Short testing sequence
- Small hardware overhead

"Universal" Approach to Concurrent Testing

Double modular redundancy



Problems: Cost, common mode faults

Solution: Parity check schemes (sacrifice fault coverage)

Main challenges:

- Identify invariant properties
- Enforce/verify invariants using small hardware overhead
- Combat finite precision effects

Stochastic Monitoring

Given data: Stochastic characterization of input
($\Rightarrow$ exact input sequence not required, i.e. monitoring)

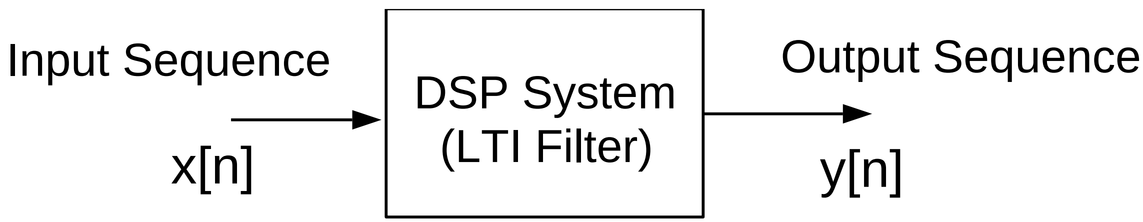
Expect: Certain stochastic properties at output

Goal: Changes in output stochastic behavior $\Rightarrow$ Fault detection/identification

Key challenge: Identify statistical invariants, match with faults

Related Work: Stochastic testing of FSM's (ACC 01)

Stochastic Monitoring/Testing of DSP Systems



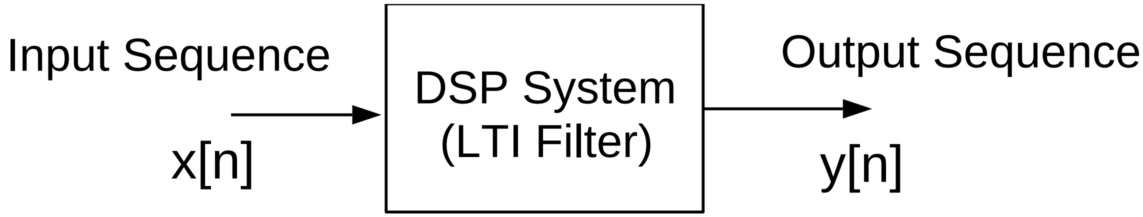
$x[n]$ Wide Sense Stationary $E(X[n]) = \mu_x$ $E(X[n+k]X[n]) = R_{xx}[k]$	$\Rightarrow$ Filter $\Rightarrow$	$y[n]$ also WSS $\mu_y = \mu_x H(0)$ $R_{yy}[k] = R_{hh}[k] * R_{xx}[k]$

Goal: Use μ_y and $R_{yy}[k]$ to detect/identify faults

Focus on: Mean $\boxed{E(Y[n])}$ and average power $\boxed{E(Y^2[n]) = R_{yy}[0]}$

More generally: (i) Higher order statistics
(ii) Other types of random processes

Example: White Gaussian WSS Input

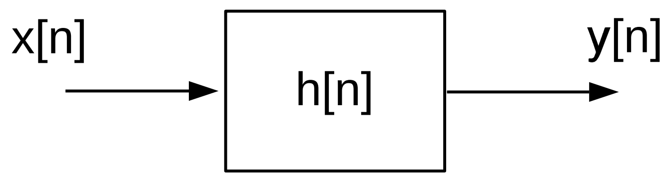


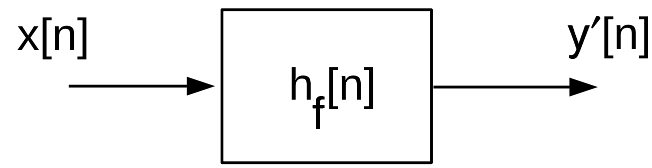
$x[n]$ White Gaussian WSS		$y[n]$ Gaussian WSS
$E(X[n]) = \mu_x$	$\Rightarrow \quad h[n] \quad \Rightarrow$	$E(Y[n]) = \mu_x H(0)$
$R_{xx}[k] = \mu_x^2 + p^2 \delta[k]$		$R_{yy}[k] = \mu_y^2 + p^2 R_{hh}[k]$

Mean: $\boxed{\mu_y = \mu_x H(0)}$ where $H(0) = \sum_{j=-\infty}^{+\infty} h[j]$

Autocorrelation: $\boxed{R_{yy}[k] = \mu_y^2 + p^2 R_{hh}[k]}$ where $R_{hh}[k] = h[k] * h[-k]$

PERMANENT FAULTS





Fault-free response:

Mean:

$\mu_y = \mu_x H(0)$

Autocorrelation:

$R_{yy}[k] = R_{hh}[k] * R_{xx}[k]$

Faulty response: $h[n] + c\delta[n - n_0]$

Mean:

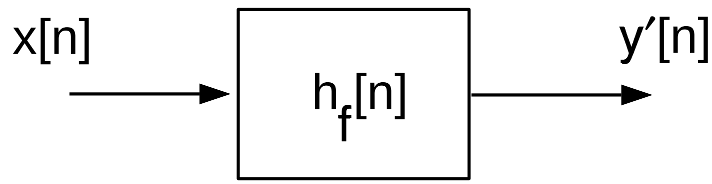
$\mu_{y'} = \mu_y + c\mu_x$

Autocorrelation:

$$\begin{aligned} R_{y'y'}[k] &= R_{yy}[k] + R_{xx}[k] * (c^2\delta[k]+ \\ &\quad ch[k+n_0] + ch[-k-n_0]) \end{aligned}$$

Focus: Changes in the *mean* of output process
(average power requires fourth moments to be stationary)

Example: White Gaussian WSS Input



Input $x[n]$**:** Gaussian Random Process

Mean: μ_x

Autocorrelation: $R_{xx}[k] = \mu_x^2 + p^2\delta[k]$

Fault-free output $y[n]$**:**

Mean: $\mu_y = \mu_x H(0)$

Average power: $E(Y^2[n]) = \mu_y^2 + p^2 R_{hh}[0] = \mu_y^2 + p^2 \sum_{j=0}^{M-1} h^2[j]$

Faulty output $y'[n]$**:**

Mean: $\mu_{y'} = \mu_y + c\mu_x$

Average power: $E((Y'[n])^2) = E(Y^2[n]) + (c^2 + c(h[n_0] + h[-n_0]))p^2$

Estimators for the Means

Estimator for input mean:

$$\hat{\mu}_x = \frac{1}{N} \sum_{j=0}^{N-1} x[j]$$

Variance:

$$\mathrm{Var}(\hat{\mu}_x) = \frac{1}{N^2} \sum_{i=-(N-1)}^{N-1} (N - |i|) C_{xx}[i]$$

Estimator for output mean:

$$\hat{\mu}_y = \frac{1}{N} \sum_{j=0}^{N-1} y[j]$$

Variance:

$$\mathrm{Var}(\hat{\mu}_y) = \frac{1}{N^2} \sum_{i=-(N-1)}^{N-1} (N - |i|) C_{yy}[i]$$

Example (Continued)

Input $x[n]$: Gaussian Random Process

Mean: μ_x

Autocorrelation: $R_{xx}[k] = \mu_x^2 + p^2\delta[k]$

Estimator $\hat{\mu}_x$ is a Gaussian random variable:

Mean: μ_x

Variance: $(\sigma_{\hat{\mu}_x})^2 = \dfrac{1}{N^2} \times N \times p^2 = \dfrac{p^2}{N}$

Estimator $\hat{\mu}_y$ is a Gaussian random variable:

Mean: μ_y

Variance: $(\sigma_{\hat{\mu}_y})^2 = \dfrac{p^2}{N^2}\left(-R_{hh}[0] + 2\sum_{i=0}^{N-1}(N-i)R_{hh}[i]\right)$

Choice of Threshold — Probabilities of Miss and False Alarm

Threshold test:

$$\boxed{\begin{array}{c} H(0)\hat{\mu}_x - T < \hat{\mu}_y < H(0)\hat{\mu}_x + T \\ \\ \Leftrightarrow \\ \\ \text{Declare No fault} \end{array}}$$

Threshold allows us to adjust the probabilities of *miss* and *false alarm*:

$$\boxed{P_M = \text{Probability that rule indicates no fault given fault}}$$

$$\boxed{P_{FA} = \text{Probability that rule indicates fault given no fault}}$$

When $x[n]$ is a Gaussian random process and μ_x is known exactly:

$$\Pr[\text{False Alarm}] = 2Q\left(\frac{T}{\sigma_{\hat{\mu}_y}}\right) \qquad \Pr[\text{Miss}] = Q\left(\frac{-T - c\mu_x}{\sigma'_{\hat{\mu}_y}}\right) - Q\left(\frac{T - c\mu_x}{\sigma'_{\hat{\mu}_y}}\right)$$

Error function: $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

Conclusions and Future Work

Potential Advantages:

- No input knowledge required
- Simple invariants
- Small hardware overhead

Future Work:

- Average power of output process
- Non-Gaussian WSS processes
- Bounds on length of observation window