University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2001

Problem Set 10

Noise Analysis, Matched Filtering, Pulse Modulation

Issued: Thursday, November 29th. **Due:** Never.

Reading from Haykin: Chapter 4, Sections 4.1–4.3; Chapter 3, Sections 3.1–3.7.

Problem 10.1

The signal $m(t) = A_c \cos(2\pi f_c t)$ (where A_c and f_c are constants) is corrupted by additive white Gaussian noise. More specifically, the corrupted signal $x(t)$ is given by

$$x(t) = m(t) + n(t) ,$$

where $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with zero mean and power spectral density $S_{NN}(f) = \frac{N_0}{2}$. Find an expression for the output signal-to-noise ratio after the signal $x(t) = m(t) + n(t)$ is applied to an LTI filter with impulse response $h(t) = e^{-t}u(t)$.

Problem 10.2

Consider the signal

$$s(t) = \begin{cases} A/2 , & 0 \le t \le T/2 , \\ -A/2 , & T/2 \le t \le T . \end{cases}$$

(a) Sketch the impulse response of a filter that is matched to this signal.

(b) Plot the output of the matched filter in part (a) when $s(t)$ is the input.

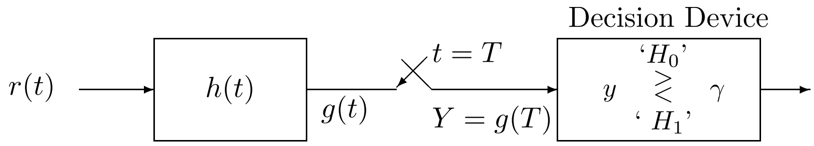
Problem 10.3

In a binary on-off signaling scheme the transmitted signal $s(t)$ is either set to zero (under hypothesis H_1), or is given by

$$s(t) = \begin{cases} A, & 0 \le t \le T , \\ 0, & \text{otherwise} \end{cases}$$

(under hypothesis H_0). The prior probabilities for the two hypotheses are given by $\Pr(H_0) = p_0$ and $\Pr(H_1) = p_1$. The signal $r(t)$ seen at the receiving end includes additive white Gaussian noise, i.e., $r(t) = s(t) + n(t)$, where $n(t)$ is a sample path of a white Gaussian random process

$N(t)$ with autocorrelation function $R_{NN}(\tau) = \frac{N_0}{2}\delta(\tau)$. In order to make a decision as to whether hypothesis H_0 or H_1 took place, we use the approach shown below.

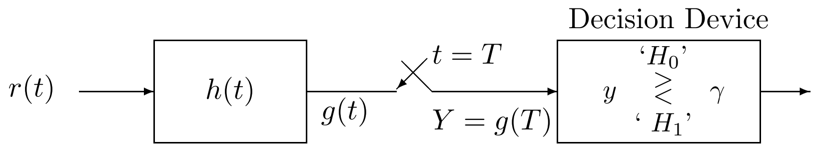


The impulse response $h(t)$ is given by $h(t) = 1$ for $0 \le t \le T$ (0 otherwise). At the decision device, the value y observed for random variable Y (i.e., the value of $g(t)$ at time $t = T$) is compared to a threshold γ. The decision is "H_0" if $y > \gamma$, and "H_1" otherwise.

(a) Find $f_{Y|H_0}(y|H_0)$, the conditional probability density of Y given hypothesis H_0.

(b) Choose γ so that the probability of error is minimized.

Problem 10.4



Consider the binary hypothesis testing situation shown above. Signal $r(t) = s(t) + n(t)$, where $s(t)$ is a deterministic signal and $n(t)$ is noise. Under hypothesis H_0, signal $s(t)$ is given by

$$s(t) = \begin{cases} \cos(\frac{\pi}{2}t)\,, & 0 \le t \le 2\,, \\ 0, & \text{otherwise.} \end{cases}$$

Under hypothesis H_1, signal $s(t)$ is zero. Assume that the prior probabilities for the two hypotheses are equal and that the noise $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with average power $N_0/2$. At the decision device, the value y observed for random variable Y is compared to a threshold γ. The decision is "H_0" if $y > \gamma$, and "H_1" otherwise.

(a) Let $T = 1$ and $h(t)$ be given by

$$h(t) = \begin{cases} 1, & 0 \le t \le T\,, \\ 0, & \text{otherwise}\,. \end{cases}$$

Find $f_{Y|H_0}(y|H_0)$ and $f_{Y|H_1}(y|H_1)$, the conditional probability densities of Y given each of the two hypotheses. Choose γ to minimize the probability of error and find the corresponding probability of error.

(b) Repeat part (a) for $T = 2$.

(c) Let $T = 2$. Choose $h(t)$ and γ so that you minimize the probability of error. What is the corresponding probability of error?

Problem 10.5

Problem 4.3 from Haykin, pp. 300–301.

Problem 10.6

Problems 3.2 and 3.3 from Haykin, p. 239.

Problem 10.7

Problem 3.14 from Haykin, p. 241.

Problem 10.8

Problems 3.17 and 3.18 from Haykin p. 242.