University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2001

Problem Set 3

Narrowband Signals Through LTI Systems, Amplitude Modulation

Issued: Thursday, Sept. 13th. **Due:** Thursday, Sept. 20th (beginning of lecture).

Reading from Haykin: Chapter 2, Sections 2.1–2.5.

Announcement: The first Mid-Semester Exam will be held on Thursday, September 27th, from 1:30pm to 2:50pm in 161 Everitt. The exam will cover all material from the beginning of the term *up to and including* the lecture on Thursday, September 20th. The corresponding material includes Problem Sets 1 through 4 and the following material from Haykin: Appendix A.2, Background and Preview and Chapter 2 (Sections 2.1–2.6).
During the exam, you can bring an 8.5×11-inch double-sided sheet of *handwritten* notes. Calculators are allowed but will not be necessary.
A copy of an old exam can be downloaded from `http://www.ece.uiuc.edu/ece359` . This exam does not necessarily resemble this year's exam (also notice that the material covered in this old exam is slightly different from the material covered in this year's exam).

Problem 3.1

The rectangular pulse

$$x(t) = \begin{cases} A\cos(2\pi f_0 t) , & 0 \le t \le T , \\ 0 , & \text{otherwise,} \end{cases}$$

goes through an LTI system with impulse response

$$h(t) = x(T-t) .$$

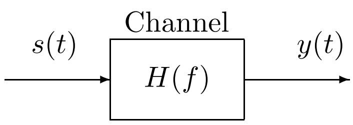
Find the output $y(t)$ of the filter under the assumption that the frequency f_0 is a large integer multiple of $\frac{1}{T}$.

Problem 3.2

Suppose the modulated signal $s(t) = m(t)\cos(2\pi f_c t)$ is applied as an input to an LTI communication channel with frequency response $H(f)$, where the modulating signal $m(t)$ is given by

$$m(t) = \text{sinc}(t/T) .$$

Assume $(1/T) = 75$ kHz and $f_c = 1300$ kHz.



(a) Make a neat and fully labeled sketch of $S(f)$.

(b) Find a time-domain expression for the output $y(t)$ of the channel if the channel frequency response is

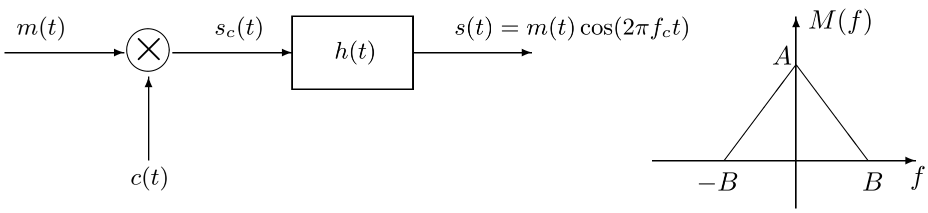
$$H(f) = e^{-j2\pi f(4\times 10^{-6})} \ .$$

(c) Find an *approximate* (but reasonably accurate) time-domain expression for the output $y(t)$ of the channel if the channel characteristics are actually as shown in Figure 3.2-1 (on the last page of the problem set) rather than as specified in (b). Also state what features of the signal and/or channel make your approximation reasonable.

Problem 3.3

The spectrum of the input signal $m(t)$ and a DSB-SC modulator are shown below. The carrier $c(t)$ available at the multiplier is *distorted* and is given by

$$c(t) = a_1 \cos(2\pi f_c t) + a_2 \cos^2(2\pi f_c t) \ .$$



(a) Determine the spectrum of the signals $s_c(t)$ and $s(t)$.

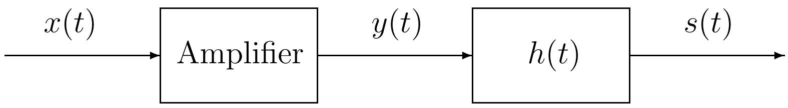
(b) What constraints should the filter $h(t)$ satisfy so that the transmitted signal $s(t)$ is the desirable one?

(c) What minimum value of f_c is required for this system to work?

Problem 3.4

A DSB-SC signal $x(t) = m(t)\cos(2\pi f_c t)$ is amplified before transmitted over a channel. Unfortunately, the amplifier is nonlinear with output $y(t)$ related to its input $x(t)$ via the relation

$$y(t) = 100x(t) + x^2(t) \ .$$



(a) Assuming the spectrum of $m(t)$ is limited to $\pm B$ Hz, find and sketch the spectra of signals $y(t)$ and $s(t)$. (Filter $h(t)$ is the standard bandpass filter used in DSB-SC modulation.)

(b) If we use coherent detection (i.e., assuming we know f_c exactly), is it possible to recover signal $m(t)$ at the receiver without distortion? If so, what are the restrictions on the value of f_c ?

Problem 3.5 (Optional)

A square-law detector is one that uses a nonlinear device to demodulate an amplitude modulated waveform. The output $y(t)$ of this nonlinear device is related to its input $x(t)$ via

$$y(t) = x(t) + \alpha x^2(t) \ ,$$

where α is a constant.
If the input to this nonlinear device is an amplitude modulated signal

$$s(t) = A_c[1 + km(t)]\cos(2\pi f_c t) \ ,$$

find (i) the output $y(t)$ and (ii) the conditions under which $m(t)$ can be recovered exactly from $y(t)$.

Problem 3.6 (Optional)

Problem 2.7 from Haykin, p. 168.

Problem 3.7

Problem 2.8 from Haykin, pp. 168–169.

Figure 3.2-1: Filter characteristics for Problem 3.2.