University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2002

Problem Set 10

Noise Analysis in Analog Modulation Schemes, Signal Detection in Noise

Issued: Thursday, November 21st. **Due:** Thursday, Dec. 5th (beginning of lecture).

Reading from Proakis (2nd Edition): Chapter 5, Chapter 7, Section 7.5 (up to and including Section 7.5.2), and lecture notes on signal detection in noise and matched filtering.

Problem 10.1

Problem 5.11, Parts 1 and 2 *only*, from Proakis (2nd Edition), p. 264.

Problem 10.2

In this problem we are interested in comparing the performance of a DSB-SC modulation scheme with the performance of an FM modulation scheme. In both cases, the message signal is $m(t) = A_m \cos 2\pi f_m t$ and the modulated signal ($s_{DSB}(t)$ or $s_{FM}(t)$) is corrupted by additive white Gaussian noise so that the received signal $r(t)$ is given by

$$r(t) = s_{DSB}(t) + n(t) \quad \text{or} \quad r(t) = s_{FM}(t) + n(t) \; .$$

The noise $n(t)$ is a sample path from a white Gaussian random process $N(t)$ with zero mean and power spectral density $S_{NN}(f) = \frac{N_0}{2}$. The following parameters are given:

$$A_m = 1 \text{ Volt}, \quad f_m = 10 \text{ KHz}, \quad N_0 = 10^{-5} \text{ W/Hz}.$$

For the purposes of this problem, you can take the signal to noise ratio at the output $y(t)$ of the demodulator to be

$$\text{SNR} = \frac{\text{Power of component of } y(t) \text{ that is due to } m(t)}{\text{Average power of component of } y(t) \text{ that is due to } n(t)} \; .$$

(a) The message signal $m(t)$ is DSB-SC modulated so that the transmitted signal $s_{DSB}(t)$ is given by

$$s_{DSB}(t) = A_d \, m(t) \cos 2\pi f_c t$$

for $f_c = 50$ MHz and $A_d = 10$. The received signal $r(t)$ is demodulated using a coherent demodulator as discussed in class. What is the signal to noise ratio at the output of the coherent demodulator?

(b) The message signal $m(t)$ is frequency modulated so that the transmitted signal $s_{FM}(t)$ is given by

$$s_{FM}(t) = A_f \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]$$

with $f_c = 50$ MHz and $k_f = 30$ KHz/Volt.

(i) Use Carson's rule to estimate the bandwidth required by $s_{FM}(t)$.

(ii) Let $A_f = 5$ Volt. If the received signal $r(t) = s_{FM}(t) + n(t)$ is demodulated using a standard FM demodulation scheme like the one we studied in class, what is the signal to noise ratio at the output of the demodulator?

(iii) Choose A_f so that $s_{FM}(t)$ has the same power as $s_{DSB}(t)$ in part (a). Find the minimum required bandwidth for $s_{FM}(t)$ so that the signal to noise ratio at the output of the FM demodulator is 6 times the signal to noise ratio at the output of the coherent DSB demodulator.

Problem 10.3

Consider the signal

$$s(t) = \begin{cases} A/2\ , & 0 \le t \le T/2\ , \\ -A/2\ , & T/2 \le t \le T\ . \end{cases}$$

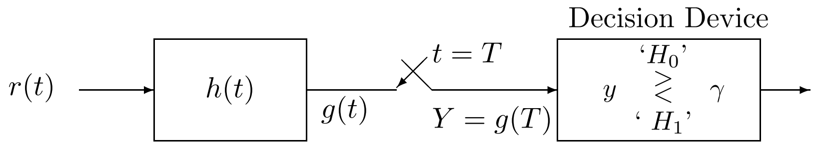
Sketch the impulse response of a filter that is matched to this signal and plot the output of the matched filter in part (a) when $s(t)$ is the input. What is the maximum value of the output?

Problem 10.4

In a binary on-off signaling scheme the transmitted signal $s(t)$ is either set to zero (under hypothesis H_1), or is given by

$$s(t) = \begin{cases} A, & 0 \le t \le T\ , \\ 0, & \text{otherwise} \end{cases}$$

(under hypothesis H_0). The prior probabilities for the two hypotheses are given by $\Pr(H_0) = p_0$ and $\Pr(H_1) = p_1$. The signal $r(t)$ seen at the receiving end includes additive white Gaussian noise, i.e., $r(t) = s(t) + n(t)$, where $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with autocorrelation function $R_{NN}(\tau) = \frac{N_0}{2}\delta(\tau)$. In order to make a decision as to whether hypothesis H_0 or H_1 took place, we use the approach shown below.

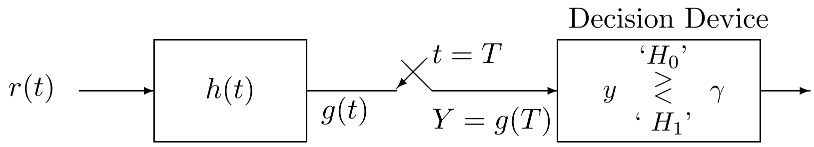


The impulse response $h(t)$ is given by $h(t) = 1$ for $0 \le t \le T$ (0 otherwise). At the decision device, the value y observed for random variable Y (i.e., the value of $g(t)$ at time $t = T$) is compared to a threshold γ. The decision is "H_0" if $y > \gamma$, and "H_1" otherwise.

(a) Find $f_{Y|H_0}(y|H_0)$, the conditional probability density of Y given hypothesis H_0.

(b) Choose γ so that the probability of error is minimized.

Problem 10.5



Consider the binary hypothesis testing situation shown above. Signal $r(t) = s(t) + n(t)$, where $s(t)$ is a deterministic signal and $n(t)$ is noise. Under hypothesis H_0, signal $s(t)$ is given by

$$s(t) = \begin{cases} \cos(\frac{\pi}{2}t)\ , & 0 \leq t \leq 2\ , \\ 0, & \text{otherwise.} \end{cases}$$

Under hypothesis H_1, signal $s(t)$ is zero. Assume that the prior probabilities for the two hypotheses are equal and that the noise $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with average power $N_0/2$. At the decision device, the value y observed for random variable Y is compared to a threshold γ. The decision is "H_0" if $y > \gamma$, and "H_1" otherwise.

(a) Let $T = 1$ and $h(t)$ be given by

$$h(t) = \begin{cases} 1, & 0 \leq t \leq T\ , \\ 0, & \text{otherwise}\ . \end{cases}$$

Find $f_{Y|H_0}(y|H_0)$ and $f_{Y|H_1}(y|H_1)$, the conditional probability densities of Y given each of the two hypotheses. Choose γ to minimize the probability of error and find the corresponding probability of error.

(b) Repeat part (a) for $T = 2$.

(c) Let $T = 2$. Choose $h(t)$ and γ so that you minimize the probability of error. What is the corresponding probability of error?