University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2002

# Problem Set 2

## Hilbert Transform, Bandpass Signals, Narrowband Signals through LTI Systems

**Issued:** Thursday, Sept. 12th. **Due:** Thursday, Sept. 19th (beginning of lecture).

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**Reading from Proakis (2nd Edition):** Chapter 2, Section 2.5.

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### Problem 2.1

Problem 2.52 from Proakis (2nd Edition), p. 67.

### Problem 2.2

Let $m(t) = \text{sinc}(t)$ and let $\widehat{m}(t)$ denote its Hilbert transform. Define

$$x(t) = m(t)\cos 2\pi f_c t - \widehat{m}(t)\sin 2\pi f_c t$$

to be a bandpass signal ($f_c >> \frac{1}{2}$).

(a) Find the pre-envelope $x_+(t)$ and the complex envelope $\widetilde{x}(t)$ of signal $x(t)$.

(b) Determine the Fourier transform and bandwidth of $x(t)$.

### Problem 2.3

(a) Show that the complex envelope of the sum of two narrowband signals (with the same carrier frequency) is equal to the sum of their individual complex envelopes.

(b) Consider a signal of the form

$$s(t) = c(t)m(t) \ ,$$

where $m(t)$ is a low-pass signal whose Fourier transform $M(f)$ is zero for $|f| > W$, and $c(t)$ is a high-pass signal whose Fourier transform $C(f)$ is zero for $|f| < W$. Show that the Hilbert transform of $s(t)$ is given by

$$\hat{s}(t) = \hat{c}(t)m(t) \ ,$$

where $\hat{c}(t)$ is the Hilbert transform of $c(t)$.

## Problem 2.4 (Optional)

Problem 2.56 from Proakis (2nd Edition), p. 68.

## Problem 2.5

The rectangular pulse

$$x(t) = \begin{cases} A\cos(2\pi f_c t)\ , & 0 \le t \le T\ , \\ 0\ , & \text{otherwise,} \end{cases}$$

goes through an LTI system with impulse response

$$h(t) = x(T - t)\ .$$

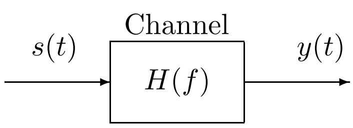
Find the output $y(t)$ of the filter under the assumption that the frequency $f_c$ is a large integer multiple of $\frac{1}{T}$.

## Problem 2.6

Suppose the modulated signal $s(t) = m(t)\cos(2\pi f_c t)$ is applied as an input to an LTI communication channel with frequency response $H(f)$, where the modulating signal $m(t)$ is given by

$$m(t) = \text{sinc}(t/T)\ .$$

Assume $(1/T) = 75$ kHz and $f_c = 1300$ kHz.




(a) Make a neat and fully labeled sketch of $S(f)$.

(b) Find a time-domain expression for the output $y(t)$ of the channel if the channel frequency response is

$$H(f) = e^{-j2\pi f(4\times 10^{-6})}\ .$$

(c) Find an *approximate* (but reasonably accurate) time-domain expression for the output $y(t)$ of the channel if the channel characteristics are actually as shown in Figure 2.6-1 (on the last page of the problem set) rather than as specified in Part (b). Also state what features of the signal and/or channel make your approximation reasonable.

Figure 2.6-1: Filter characteristics for Problem 2.6.