University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2002

Problem Set 9

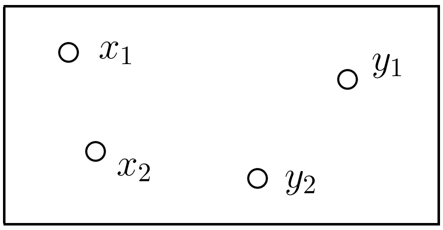
White Noise, Narrow-Band Noise, Noise Analysis in Analog Modulation Schemes

Issued: Thursday, Nov. 15th. **Due:** Thursday, Nov. 22nd (beginning of lecture).

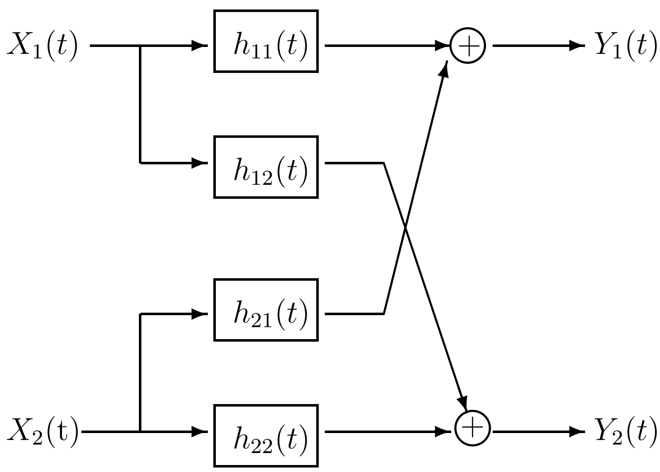
Reading from Proakis (2nd Edition): Chapter 4, Sections 4.5 and 4.6, and Chapter 5, Section 5.1.

Problem 9.1

In a variety of real communication situations two (or more) uncorrelated sources are received through channels or systems with crosstalk. Since signals interfere with each other, it is essential that they are separated at the receiving end. One such scenario is shown below, where we have two sources, denoted by x_1 and x_2, and two receivers, denoted by y_1 and y_2.



The above scenario is modeled in terms of LTI systems as shown below. The impulse response from source i to receiver j is denoted by $h_{ij}(t)$.



Approaches for recovering the signals from sources x_1 and x_2 usually involve estimating the impulse responses $h_{ij}(t)$. In this problem we investigate such techniques; to simplify the problem we assume that $h_{11}(t) = h_{22}(t) = \delta(t)$. We also assume that $h_{12}(t)$ and $h_{21}(t)$ are stable and causal and that the sources can be modeled by uncorrelated wide-sense stationary random processes $X_1(t)$ and $X_2(t)$ (with zero mean and known autocorrelation functions $R_{X_1X_1}(\tau)$ and $R_{X_2X_2}(\tau)$).

(a) Show that $Y_1(t)$ and $Y_2(t)$ are jointly wide-sense stationary and determine $R_{Y_1Y_1}(\tau)$, $R_{Y_2Y_2}(\tau)$ and $R_{Y_1Y_2}(\tau)$ in terms of $R_{X_1X_1}(\tau)$, $R_{X_2X_2}(\tau)$, $h_{12}(t)$ and $h_{21}(t)$.

(b) Suppose that $h_{12}(t) = h_{21}(t) = h(t)$ and that you can measure *only one* of $R_{Y_1Y_1}(\tau)$, $R_{Y_2Y_2}(\tau)$ or $R_{Y_1Y_2}(\tau)$. Which one would be most helpful in determining $h_{12}(t)$?

Problem 9.2 (Optional)

A random process $X(t)$ is defined as

$$X(t) = A\cos(2\pi f_c t) \ ,$$

where A is a Gaussian random variable with zero mean and variance σ_A^2. This random process is applied to an ideal integrator producing the output random process

$$Y(t) = \int_0^t X(\tau)d\tau \ .$$

(a) Determine the pdf of the random variable $Y = Y(t_1)$ for some time instant $t_1 > 0$.

(b) Is $Y(t)$ stationary?

(c) Is $Y(t)$ ergodic?

Problem 9.3 (Optional)

Problem 4.69 from Proakis (2nd Edition), p. 215.

Problem 9.4

Problem 4.73 from Proakis (2nd Edition), p. 216.

Problem 9.5 (Optional)

Consider a narrow-band Gaussian noise process $N(t)$ with zero mean and power spectral density

$$S_{NN}(f) = \begin{cases} \frac{N_0}{2} \ , & f_c - B \le f \le f_c + B \ , \quad -f_c - B \le f \le -f_c + B \ , \\ 0 \ , & \text{otherwise.} \end{cases}$$

Find the probability density function of a sample of the envelope of $n(t)$, i.e., find $f_E(e)$ of the random variable $E = c(t)$, where $c(t)$ is the envelope of the band-limited random process $N(t)$.

Problem 9.6

Problem 5.1 from Proakis (2nd Edition), p. 261.

Problem 9.7

The signal $m(t) = A_c \cos(2\pi f_c t)$ (where A_c and f_c are constants) is corrupted by additive white Gaussian noise. More specifically, the corrupted signal $x(t)$ is given by

$$x(t) = m(t) + n(t) \ ,$$

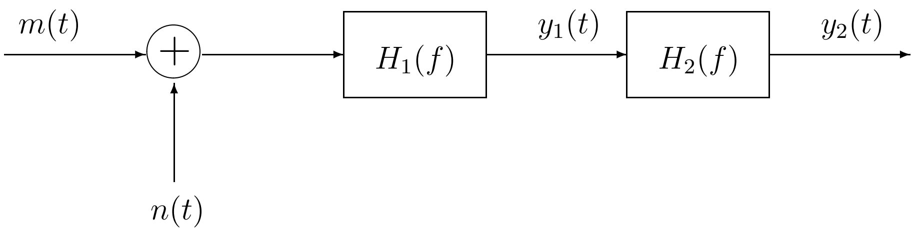
where $n(t)$ is a sample path of a white Gaussian random process $N(t)$ with zero mean and power spectral density $S_{NN}(f) = \frac{N_0}{2}$. Find an expression for the output signal-to-noise ratio after the signal $x(t) = m(t) + n(t)$ is applied to an LTI filter with impulse response $h(t) = e^{-t}u(t)$.

Problem 9.8

Problem 5.3 from Proakis (2nd Edition), pp. 261–262.

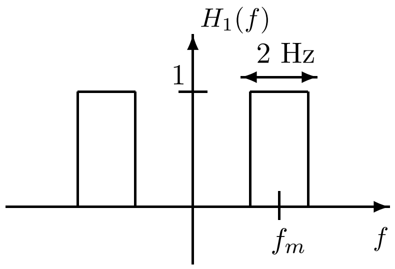
Problem 9.9

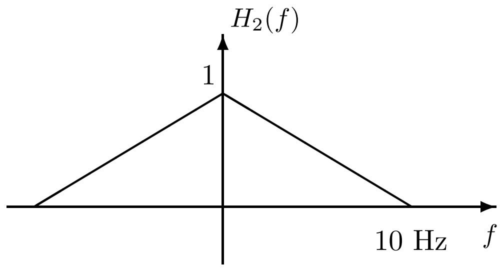
The message signal $m(t) = \cos(2\pi f_m t)$ gets corrupted by additive white Gaussian noise $N(t)$ with zero mean and power spectral density $S_{NN}(f) = 10^{-1}$ W/Hz. The resulting signal gets filtered by the cascade of the two filters shown below.



The frequency response of the filters is given by

$$H_1(f) = \begin{cases} 1, & f_m - 1 \text{ Hz} < |f| < f_m + 1 \text{ Hz} \ , \\ 0, & \text{otherwise.} \end{cases} \qquad H_2(f) = \begin{cases} 1 - \frac{|f|}{10}, & |f| < 10 \text{ Hz} \ , \\ 0, & \text{otherwise.} \end{cases}$$





(a) For this part, assume that $f_m = 5$ Hz.

(i) Find the signal to noise ratio at the output $y_1(t)$ of filter $H_1(f)$.

(ii) Find the signal to noise ratio at the output $y_2(t)$ of filter $H_2(f)$.

(b) Repeat part (a) for $f_m = 9$ Hz.

Note: The signal to noise ratio is defined as

$$\text{SNR} = \frac{\text{Power of component of } y(t) \text{ that is due to } m(t)}{\text{Average power of component of } y(t) \text{ that is due to } n(t)}\,.$$