University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 459: COMMUNICATIONS I

Fall 2004

Problem Set 2

Hilbert Transform, Bandpass Signals, Narrowband Signals through LTI Systems, Group and Phase Delays

Issued: Thursday, Sept. 9th. **Due:** Tuesday, Sept. 21st (beginning of lecture).

Reading from Lathi: Chapters 2 and 3.

Reading from Haykin: Chapter 1 and Appendix A.2.

Problem 2.1 (Optional)

(a) Problem 3.3-6 from Lathi, p. 146.

(b) Problem 3.3-7 from Lathi, p. 146.

Problem 2.2

(a) Show that the complex envelope of the sum of two narrowband signals (with the same carrier frequency) is equal to the sum of their individual complex envelopes.

(b) Consider a signal of the form

$$s(t) = c(t)m(t) ,$$

where $m(t)$ is a lowpass signal whose Fourier transform $M(\omega)$ is zero for $|\omega| > W$, and $c(t)$ is a highpass signal whose Fourier transform $C(\omega)$ is zero for $|\omega| < W$. Show that the Hilbert transform of $s(t)$ is given by

$$\hat{s}(t) = \hat{c}(t)m(t) ,$$

where $\hat{c}(t)$ is the Hilbert transform of $c(t)$.

Problem 2.3

Let $m(t) = \text{sinc}(t)$ and let $\widehat{m}(t)$ denote its Hilbert transform. Define

$$x(t) = m(t)\cos\omega_c t - \widehat{m}(t)\sin\omega_c t$$

to be a bandpass signal (assume $\omega_c >> \pi$).

(a) Find the pre-envelope $x_+(t)$ and the complex envelope $\tilde{x}(t)$ of signal $x(t)$.

(b) Determine the Fourier transform and bandwidth of $x(t)$.

Problem 2.4

Consider the following *demodulation* process. A lowpass signal $g(t)$ with bandwidth W is recovered from the modulated signal $s(t) = g(t)\cos(\omega_c t)$ (with *carrier frequency* ω_c) via the following process:

- Multiplication of $s(t)$ by $2\cos(\omega_c t)$ to obtain $r(t)$.
- Lowpass filtering of $r(t)$ with an ideal lowpass filter of bandwidth W.

Show that this demodulation process will be successful in recovering $g(t)$ as long as $W < \omega_c$.

Problem 2.5

The rectangular pulse

$$x(t) = \begin{cases} A\cos(\omega_0 t)\,, & 0 \le t \le T \\ 0\,, & \text{otherwise} \end{cases}$$

goes through an LTI system with impulse response

$$h(t) = x(T - t)\ .$$

Find the output $y(t)$ of the filter under the assumption that the frequency ω_0 is a large integer multiple of $\frac{2\pi}{T}$.

Problem 2.6 (Optional)

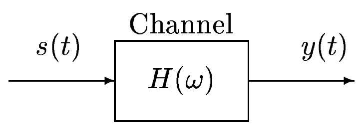
Problem 3.5-4 from Lathi, p. 148 (assume that distortionless transmission requires an amplitude variation of less than 4% and a time delay variation of less than 2%).

Problem 2.7

Suppose the modulated signal $s(t) = m(t)\cos(\omega_c t)$ is applied as an input to an LTI communication channel with frequency response $H(\omega)$, where the modulating signal $m(t)$ is given by

$$m(t) = \operatorname{sinc}\left(\frac{\pi t}{T}\right)\ .$$

Assume $(1/T) = 75$ kHz and $\omega_c = 2\pi \times 1300$ kHz.



(a) Make a neat and fully labeled sketch of $S(\omega)$.

(b) Find a time-domain expression for the output $y(t)$ of the channel if the channel frequency response is

$$H(\omega) = e^{-j\omega(4\times10^{-6})} .$$

(c) Find an *approximate* (but reasonably accurate) time-domain expression for the output $y(t)$ of the channel if the channel characteristics are (not as specified in part (b) but are) known to satisfy

$$\begin{aligned} |H(\omega_c/2)| &= 1 \\ \angle H(\omega_c/2) &= -\tfrac{\pi}{2} \\ \tau_g(\omega_c/2) &= 1.2\text{sec} \end{aligned} \qquad \begin{aligned} |H(\omega_c)| &= 1 \\ \angle H(\omega_c) &= \tfrac{\pi}{2} \\ \tau_g(\omega_c) &= 1\text{sec} \end{aligned} \qquad \begin{aligned} |H(2\omega_c)| &= 1 \\ \angle H(2\omega_c) &= \tfrac{3\pi}{4} \\ \tau_g(2\omega_c) &= 0.5\text{sec} \end{aligned}$$

where $\tau_g(\omega)$ denotes the group delay at frequency ω.

Also state what features of the signal and/or channel make your approximation reasonable.