University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 459: COMMUNICATIONS I

Fall 2005

Problem Set 2

Hilbert Transform, Bandpass Signals, Narrowband Signals through LTI Systems

Issued: Thursday, Sept. 8th. **Due:** Thursday, Sept. 15th (beginning of lecture).

Reading from Lathi: Chapters 2 and 3.

Announcement: The two mid-semester exams have been scheduled for:
(i) October 6, 7:00-9:00pm, room 165EL (NOTE THE TIME CHANGE),
(ii) November 10, 7:00-9:00pm, room 165EL (NOTE THE TIME CHANGE).
Mid-semester exams will be closed book; one double-sided sheet of notes (handwritten, 8 1/2" x 11") will be allowed for the first mid-semester exam; two double-sided sheets of notes (handwritten, 8 1/2" x 11") will be allowed for the second mid-semester exam.

Problem 2.1

(a) Find the half-power bandwidth of the signal $x(t)$ given by

$$x(t) = \delta(t) + e^{-2t}u(t) \ .$$

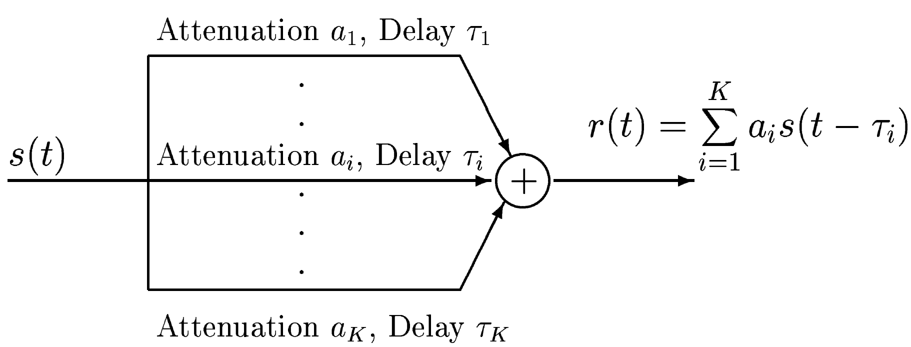
(b) Find the energy of the signal $x(t)$ given by

$$x(t) = e^{-\alpha t}u(t) - e^{-\beta t}u(t)$$

for $\alpha > \beta > 0$.

Problem 2.2

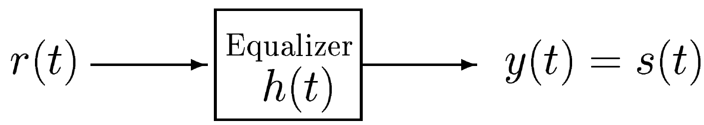
In wireless communications, the signal arriving at the receiver frequently suffers *multipath distortion*. More specifically, the received signal $r(t)$ is the superposition of multiple versions of the transmitted signal $s(t)$, each of which is attenuated and delayed by a different amount.



For the purposes of this problem, assume that the received signal $r(t)$ is given by

$$r(t) = s(t) + \frac{1}{4}s(t-T) \ ,$$

where $s(t)$ is the transmitted signal and $T > 0$ is a positive delay. The signal $r(t)$ is processed by a linear time-invariant (LTI) system with impulse response $h(t)$ so that its output $y(t)$ is the transmitted signal $s(t)$. This system is called an *equalizer*.



(a) Find the frequency response $H(\omega)$ of the equalizer and determine its phase as a function of ω.

(b) The impulse response of the equalizer can be expressed in the form

$$h(t) = \sum_{k=0}^{+\infty} h_k \delta(t-kT) \ .$$

Find h_0, h_1 and h_2.

Hint: Recall that for $|x| < 1$, the following identity is true: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$

Problem 2.3 (Optional)

(a) Problem 3.3-6 from Lathi, p. 146.

(b) Problem 3.3-7 from Lathi, p. 146.

Problem 2.4

(a) Show that the complex envelope of the sum of two narrowband signals (with the same carrier frequency) is equal to the sum of their individual complex envelopes.

(b) Consider a signal of the form

$$s(t) = c(t)m(t) \ ,$$

where $m(t)$ is a lowpass signal whose Fourier transform $M(\omega)$ is zero for $|\omega| > W$, and $c(t)$ is a highpass signal whose Fourier transform $C(\omega)$ is zero for $|\omega| < W$. Show that the Hilbert transform of $s(t)$ is given by

$$\hat{s}(t) = \hat{c}(t)m(t) \ ,$$

where $\hat{c}(t)$ is the Hilbert transform of $c(t)$.

Problem 2.5

Let $m(t) = \text{sinc}(t)$ and let $\widehat{m}(t)$ denote its Hilbert transform. Define

$$x(t) = m(t)\cos\omega_c t - \widehat{m}(t)\sin\omega_c t$$

to be a bandpass signal (assume $\omega_c >> \pi$).

(a) Find the pre-envelope $x_+(t)$ and the complex envelope $\widetilde{x}(t)$ of signal $x(t)$.

(b) Determine the Fourier transform and bandwidth of $x(t)$.

Problem 2.6

Consider the following *demodulation* process. A lowpass signal $g(t)$ with bandwidth W is recovered from the modulated signal $s(t) = g(t)\cos(\omega_c t)$ (with *carrier frequency* ω_c) via the following process:

- Multiplication of $s(t)$ by $2\cos(\omega_c t)$ to obtain $r(t)$.
- Lowpass filtering of $r(t)$ with an ideal lowpass filter of bandwidth W.

Show that this demodulation process will be successful in recovering $g(t)$ as long as $W < \omega_c$.

Problem 2.7

The rectangular pulse

$$x(t) = \begin{cases} A\cos(\omega_0 t)\ , & 0 \le t \le T \\ 0\ , & \text{otherwise} \end{cases}$$

goes through an LTI system with impulse response

$$h(t) = x(T - t)\ .$$

Find the output $y(t)$ of the filter under the assumption that the frequency ω_0 is a large integer multiple of $\frac{2\pi}{T}$.