University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 459: Communications I

Fall 2005

Problem Set 4

Angle Modulation, Phase and Frequency Modulation

Issued: Thursday, Sept. 22nd. **Due:** Tuesday, October 4th (beginning of lecture).

Reading from Lathi: Chapter 5 (excluding Section 5.5).

Announcement: The first Mid-Semester Exam will be held on Thursday, October 6th, from 7-9pm in 165 Everitt. The exam will cover all material from the beginning of the term *up to and including* the lecture on Tuesday, September 27th. The corresponding material includes Problem Sets 1 through 4 and Chapters 1, 2, 3, 4 (excluding Sections 4.7–4.9) and 5 (excluding Section 5.5) from Lathi.

During the exam, you can bring an 8.5 × 11-inch double-sided sheet of *handwritten* notes. Calculators are allowed but will not be necessary.

A copy of an old exam is available for download from the course website. This exam does not necessarily resemble this year's exam (also notice that the time allowed and the material covered in this old exam are slightly different from this year's exam).

Problem 4.1

(a) Problem 5.1-1 from Lathi, p. 248.

(b) Problem 5.2-2 from Lathi, p. 249.

Problem 4.2

Consider the FM signal

$$s(t) = A_c \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\tau) d\tau\right] ,$$

where $m(t)$ is the tone signal $m(t) = A_m \sin(\omega_m t)$. The following parameters are given:

$$A_c = 200V, \quad \omega_c = 160\pi \text{ Mrad/s}, \quad k_f = 20\pi \text{ Krad/(s Volt)}, \quad A_m = 1V, \quad \omega_m = 16\pi \text{ Krad/s} .$$

(a) Find the frequency deviation $\Delta\omega$.

(b) Find the modulation index β.

(c) The narrowband FM signal approximation uses

$$s_e(t) = A_c \cos(\omega_c t) - A_c \left[k_f \int_{-\infty}^{t} m(\tau) d\tau\right] \sin(\omega_c t)$$

as an approximation to $s(t)$. Sketch $|S_e(\omega)|$ and find its bandwidth.

(d) Find the power of $s_e(t)$ and compare it to the power of $s(t)$.

Problem 4.3

Problem 5.2-6 from Lathi, p. 249.

Problem 4.4 (Optional)

Recall that the complex envelope $\widetilde{s}(t)$ of a real signal $s(t)$ is generally complex and can be written in the form

$$\widetilde{s}(t) = s_I(t) + js_Q(t) ,$$

where $s_I(t)$ and $s_Q(t)$ are the in-phase and quadrature components of $s(t)$.
Find the expressions for the in-phase and quadrature components of the following signals:

(a) PM signal $s(t) = A\cos[\omega_c t + k_p m(t)]$;

(b) FM signal $s(t) = A\cos[\omega_c t + k_f \int_0^t m(t)dt]$.

Problem 4.5

The sinusoidal message signal $m(t) = A_m \cos(\omega_m t)$ is frequency modulated so that the transmitted signal $s(t)$ is given by

$$\begin{aligned} s(t) &= A_c \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\tau)d\tau\right] \\ &= A_c \cos\left[\omega_c t + \beta \sin \omega_m t\right] , \end{aligned}$$

where $\beta = \dfrac{k_f A_m}{\omega_m}$. Given that

$$\omega_c = 20 \text{ Mrad/s}, \quad A_m = 10V, \quad k_f = 6 \text{ Krad/s/V} ,$$

specify the range of possible ω_m such that the bandwidth of the resulting $s(t)$ is below 200 Krad/s.

Problem 4.6 (Optional)

Problem 5.4-2 from Lathi, p. 250.