University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 434: RANDOM PROCESSES

Spring 2004

Problem Set 5

Properties of Random Processes, Independent Increments Property, Martingale Property, Markov Property, Calculus of Random Processes

Issued: Friday, April 2nd **Due:** Beginning of lecture on Monday, April 12th

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Reading from Hajek: Chapter 4; Chapter 5, Sections 5.1–5.3.

Reading from Stark and Woods: Chapter 6, Sections 6.4–6.6; Chapter 7, Sections 7.1–7.2; Chapter 8, Section 8.1.

Problem 5.1

Let X be a random variable and $Y_0, Y_1, Y_2, ...$ be a sequence of random variables. Show that the random process defined by

$$Z[n] = E[X \mid Y_0, Y_1, ..., Y_n]$$

is a Martingale.

Problem 5.2

A nonuniform Poisson counting process $N(t)$ with time varying rate $\lambda(t)$ ($\lambda(t) \geq 0$ for all $t \geq 0$) is defined for $t \geq 0$ as follows:

(i) $N(0) = 0$;

(ii) $N(t)$ has independent increments;

(iii) For all $t_2 \geq t_1$,

$$P[N(t_2) - N(t_1) = k] = \frac{u^k}{k!}\exp(-u), \quad \text{for } k \geq 0 \text{ ,}$$

where $u = \int_{t_1}^{t_2} \lambda(v)dv$.

Answer the following:

(a) Find $\mu_N(t)$.

(b) Find $R_N(t_1, t_2)$.

(c) Repeat parts (a) and (b) for the case when $\lambda(t) = 1 + 2t$.

Problem 5.3

From Hajek, Chapter 4: Problems 6 and 7.

Problem 5.4

A random telegraph signal is a random process $X(t)$ defined for $t \geq 0$ as follows: $X(0) = +1$ and $X(t)$ switches between $+1$ and -1 according to a Poisson random arrival sequence $T[n]$, i.e.

$$X(t) = \begin{cases} 1, & 0 \leq t < T[1], \\ -1, & T[1] \leq t < T[2], \\ 1, & T[2] \leq t < T[3], \\ \cdots & \cdots \end{cases}$$

Assume that the rate parameter λ of the Poisson random arrival time sequence is known.

(a) Argue that $X(t)$ is a Markov process and draw and label its state transition diagram.

(b) Find the steady-state probability that $X(t) = +1$ in terms of the rate parameter λ.

Problem 5.5

From Hajek, Chapter 4: Problems 8 and 9.

Problem 5.6

From Hajek, Chapter 5: Problems 1 and 2.