ECE 517, Fall 09 **PROBLEM SET 2** **Due Wednesday, Oct 7**

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1. Consider the class of scalar plants

$$\dot{y} = ay + bu, \qquad a \in \mathbb{R},\ b > 0 \tag{1}$$

In Section 3.1.1 of class notes, it is shown that the controller (19) is a universal regulator for this class of plants, with the help of the Lyapunov function $V(y) = y^2/2$. For the same controller, find a Lyapunov function more similar to the Lyapunov function (3) used in Example 1 (i.e., one that depends on a, b, k) such that convergence of y to 0 in closed loop can be proved by a direct application of Theorem 2.

2. Consider again the class of scalar plants (1). Show that there doesn't exist a *linear* universal regulator for this class of plants, i.e., a universal regulator of the form (22) from class notes with f and h linear functions. Here the dimension of z can be arbitrary. (Thus you cannot use the non-existence result for rational controllers proved in class, because it is restricted to scalar z.)

3. Design a universal regulator for the class of scalar plants

$$\dot{y} = a\varphi(y) + bu, \qquad a \in \mathbb{R},\ b > 0$$

where $\varphi(\cdot)$ is a fixed known function. Justify rigorously that it works.

4. Consider a linear system

$$\dot{x} = Ax + Bu$$

and assume that A is Hurwitz, so that we have $\|e^{At}\| \leq ce^{-\lambda_0 t}$ for some $c, \lambda_0 > 0$. Prove the following:

a) If $u \in L_2$ or u is bounded, then x is bounded. (Hint: use the variation-of-constants formula and the Cauchy-Schwartz and Hölder's inequalities.)

b) If $u \in L_2$ or $u \to 0$, then $x \to 0$. (Hint: use part a).)

5. Email me, separately from the homework, a brief (1-2 paragraphs) tentative description of your final project. A plain text message is preferred. Explain what the topic of your project will be and what are its main goals. (See the first lecture for project guidelines. Project presentations will start either right before or right after the Thanksgiving break.)