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1. The purpose of this exercise is to extend the backstepping procedure beyond pure integrator backstepping. Consider the system

$$\begin{aligned}\dot{x} &= f(x) + g(x)\xi \\ \dot{\xi} &= f_1(x,\xi) + g_1(x,\xi)u\end{aligned}$$

where x, ξ, u are all scalar variables and $g_1(x,\xi) \neq 0 \; \forall\, x, \xi$. Assume that a Lyapunov function $V_0(x)$ and a feedback law $k_0(x)$ (with $k_0(0) = 0$) are known such that

$$\frac{\partial V_0}{\partial x}\big(f(x) + g(x)k_0(x)\big) \leq -W(x) < 0 \qquad \forall\, x \neq 0$$

Construct a control Lyapunov function $V_1(x,\xi)$ and a stabilizing feedback law $k_1(x,\xi)$ for the 2-D system.

2. Consider the system

$$\begin{aligned}\dot{x} &= \theta x + \xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u\end{aligned}$$

where x, ξ_1, ξ_2, u are all scalar variables and θ is an unknown parameter. Continue the backstepping procedure given in Section 5.2 in the notes and design an adaptive control law that asymptotically stabilizes this system.

3. Prove Lemma 4 from the lecture notes. Be sure to give explicit expressions for λ, c in terms of α_0, T_0 and vice versa.

4. Prove that the vector signal

$$\phi(t) := \begin{pmatrix} A\sin(\omega t + \alpha) \\ \sin\omega t \end{pmatrix}$$

is PE under suitable constraints on the numbers A, ω, α (specify these constraints).

5. Solve Exercise 4.2 on page 246 in [Ioannou-Sun].[1]

6. Email me an update on how your project is going. A plain text message (no attachments) is preferred. If I had any comments on your initial project proposal, explain how you're going to address them. If there was any change in project's topic or scope, give details on that. By this homework's deadline your project must be well defined. Come to my office hours to discuss any remaining concerns.

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[1] Use the link on the class homepage to access the electronic version of [Ioannou-Sun].