ECE 517, Fall 09 **PROBLEM SET 4** **Due Wednesday, Nov 18**

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1. Solve Exercise 4.9, parts (b)–(e) on page 247 in [Ioannou-Sun]. (Assume part (a) is true.)

2. Implement the indirect MRAC control design example given in class via computer simulation. Plot the behavior of the plant state as well as control gains over time. Does loss of stabilizability ($\hat{b} \to 0$) actually happen? Investigate this in the following cases:

a) $b > 0$ and $\hat{b}$ is initialized with the correct sign ($\hat{b}(0) > 0$).

b) $b > 0$ but $\hat{b}$ is initialized with the wrong sign ($\hat{b}(0) < 0$).

In each case, if you experience loss of stabilizability, modify the update law for $\hat{b}$ to get rid of the problem. Submit both sets of plots (with and without modification) and compare them.

3. Consider the scalar system $\dot{x} = f(x) + g(x)u$ where $f(x) = -x\sin^2(x^2)$, $g(x) = \cos(x^2)$.

a) Construct a feedback law $u = k(x)$ which makes the closed-loop system GAS. Justify this using a Lyapunov function.

b) Now suppose that the state measurements available to the feedback law are affected by an additive disturbance, resulting in the system $\dot{x} = f(x) + g(x)k(x + d)$ where f, g are the same as before and k is the feedback you found in part a). Is this system ISS with respect to d? Prove or disprove.

4. Consider again the system from the previous homework:

$$\begin{aligned} \dot{x} &= \theta x + \xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u \end{aligned}$$

where x, ξ_1, ξ_2, u are all scalar variables and θ is an unknown parameter. Use the adaptive ISS backstepping procedure given in Section 7.3.1 in the notes to design a feedback law that makes the closed-loop system ISS with respect to $(\tilde{\theta}, \dot{\hat{\theta}})^T$ (for an arbitrary tuning law). Here you need to do the initialization step as well as the two recursion steps.

5. Consider the following adaptive control system.

Plant: $\dot{y} = ay + bu$, where a and $b \neq 0$ are unknown parameters.

Control law: $u = -ky$. Update law for k: $\dot{k} = \hat{b}(\hat{a} - \hat{b}k + 1)$.

(Interpretation: drive k to the equilibrium value $k = \frac{\hat{a}+1}{\hat{b}}$, but stop if $\hat{b} \to 0$ to keep k bounded.)

Estimator: $\dot{\hat{y}} = -(\hat{y} - y) + \hat{a}y + \hat{b}u$.

Update laws for $\hat{a}$, $\hat{b}$ (as in class): $\dot{\hat{a}} = -\gamma e y$, $\dot{\hat{b}} = -\gamma e u$, where $\gamma > 0$ and $e = \hat{y} - y$.

Show that there exist values of a, b and initial values of $y, \hat{y}, k, \hat{a}, \hat{b}$ for which we get a trajectory along which $e \equiv 0$ but $y \nearrow \infty$. Interpret this situation in terms of lack of detectability.

6. Email me a tentative outline of your project presentation (in a plain text message, no attachments). Indicate what points you will cover, in which order, and approximately how much time you will spend on each point. The length of your presentation (including questions) will be 20 minutes for individual projects, 30 minutes for teams of 2 people. Optionally, you may indicate your preferred days and times (the presentations will be on Wed Dec 2, Mon Dec 7, and Wed Dec 9, from 4pm to 5:45–6pm). Of course I cannot promise that I will accommodate all requests. If you don't tell me any preferences/constraints by this homework's deadline, I will assume that you are OK with speaking at any of the above times.