

Summary of Equations: 3-D 4-DOF Biped w/ Point-Feet, Midleg-Mass

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Variables

ω -- > stance yaw angle
 ϕ -- > stance roll angle
 θ_{ns} -- > swing pitch angle
 θ_s -- > stance pitch angle

■ Constants

l -- > leg length
 M -- > hip mass
 m -- > midleg mass
 γ -- > slope angle
 β -- > potential shaping angle
(difference between passive slope angle and actual slope γ)
 c -- > Lateral control proportional constant (in functional conserved quantity)
 d -- > Lateral control bias constant (in functional conserved quantity)
 r -- > Yaw control proportional constant (in functional conserved quantity)
 s -- > Yaw control bias constant (in functional conserved quantity)
 K_p -- > Lateral proportional control gain for conserved quantity surface tracking
 L_p -- > Yaw proportional control gain for conserved quantity surface tracking

Lagrangian Dynamics

$\mathbf{q} = \{\{\omega[t]\}, \{\phi[t]\}, \{\theta_{ns}[t]\}, \{\theta_s[t]\}\};$
 $\mathbf{qdot} = D[\mathbf{q}, t];$

KE = 1 / 2 Transpose[qdot].Mmatrix.qdot;

PE

$$-\frac{1}{2} g l (m \cos[\theta_{ns}[t]] - (3m + 2M) \cos[\theta_s[t]]) \cos[\phi[t]]$$

Lagrangian = KE - PE;

■ Mass/Inertia Matrix

Mmatrix

$$\left\{ \left\{ -\frac{1}{8} l^2 \left(\cos[\phi[t]]^2 (-6m - 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 4(-3m - 2M + 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right), \right. \right.$$

$$-\frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]),$$

$$\frac{1}{4} l^2 m (1 - 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]],$$

$$\left. \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right\}, \right.$$

$$\left\{ -\frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]), \right.$$

$$\left. \frac{1}{8} l^2 (6m + 4M + m \cos[2\theta_{ns}[t]] - 8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t]]), 0, 0 \right\},$$

$$\left\{ \frac{1}{4} l^2 m (1 - 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]], 0, \frac{l^2 m}{4}, -\frac{1}{2} l^2 m \cos[\theta_{ns}[t] - \theta_s[t]] \right\},$$

$$\left\{ \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]], 0, -\frac{1}{2} l^2 m \cos[\theta_{ns}[t] - \theta_s[t]], \frac{1}{4} l^2 (5m + 4M) \right\} \}$$

■ Coriolis Matrix

Dim = Dimensions[Mmatrix];

Cmatrix = Table[0, {Dim[[1]]}, {Dim[[1]]}];

For[i = 1, i ≤ Dim[[1]], i++,

For[j = 1, j ≤ Dim[[1]], j++,

Cmatrix[[i, j]] =

FullSimplify[1 / 2 Sum[(D[Mmatrix[[i, j]], q[[k, 1]]] + D[Mmatrix[[i, k]], q[[j, 1]]] - D[Mmatrix[[k, j]], q[[i, 1]]) qdot[[k, 1]], {k, Dim[[1]]}]]];

Cmatrix

$$\left\{ \left\{ \frac{1}{16} l^2 \left(4m \left(\cos[\theta_{ns}[t]] \cos[\phi[t]]^2 (\sin[\theta_{ns}[t]] - 2 \sin[\theta_s[t]]) + 2 \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_{ns}'[t] + 2 \left(-4m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] + (5m + 4M) \cos[\phi[t]]^2 \sin[2\theta_s[t]] + 4m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] \sin[\phi[t]]^2 \right) \theta_s'[t] + (6m + 4M + m \cos[2\theta_{ns}[t]] - 8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t]]) \sin[2\phi[t]] \phi'[t] \right), \right. \right.$$

$$\begin{aligned}
& \frac{1}{16} l^2 (4 m \cos[\phi[t]] \sin[\theta_{ns}[t]] (\sin[\theta_{ns}[t]] - 2 \sin[\theta_s[t]]) \theta_{ns}'[t] + \\
& \quad 4 \cos[\phi[t]] \sin[\theta_s[t]] (-2 m \sin[\theta_{ns}[t]] + (5 m + 4 M) \sin[\theta_s[t]]) \theta_s'[t] + \\
& \quad 2 (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\phi[t]] \phi'[t] + \\
& \quad (6 m + 4 M + m \cos[2 \theta_{ns}[t]] - 8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]]) \\
& \quad \sin[2 \phi[t]] \omega'[t]), \\
& \frac{1}{4} l^2 m (\cos[\phi[t]] (\sin[\theta_{ns}[t]] - 2 \sin[\theta_s[t]]) (\sin[\theta_{ns}[t]] \phi'[t] + \cos[\theta_{ns}[t]] \cos[\phi[t]] \omega'[t]) + \\
& \quad 2 \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] (\theta_{ns}'[t] + \sin[\phi[t]] \omega'[t])), \\
& \frac{1}{8} l^2 (-4 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t] + 2 \cos[\phi[t]] \sin[\theta_s[t]] \\
& \quad (-2 m \sin[\theta_{ns}[t]] + (5 m + 4 M) \sin[\theta_s[t]]) \phi'[t] + (-4 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] + \\
& \quad (5 m + 4 M) \cos[\phi[t]]^2 \sin[2 \theta_s[t]] + 4 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] \sin[\phi[t]]^2) \omega'[t]), \\
& \left\{ -\frac{1}{8} l^2 \cos[\phi[t]] (2 m \cos[\theta_{ns}[t]] (\cos[\theta_{ns}[t]] - 2 \cos[\theta_s[t]]) \theta_{ns}'[t] + \right. \\
& \quad 2 \cos[\theta_s[t]] (-2 m \cos[\theta_{ns}[t]] + (5 m + 4 M) \cos[\theta_s[t]]) \theta_s'[t] + \\
& \quad (6 m + 4 M + m \cos[2 \theta_{ns}[t]] - 8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \\
& \quad \sin[\phi[t]] \omega'[t]), -\frac{1}{4} l^2 (m (\cos[\theta_{ns}[t]] - 2 \cos[\theta_s[t]]) \sin[\theta_{ns}[t]] \theta_{ns}'[t] + \\
& \quad (-2 m \cos[\theta_{ns}[t]] + (5 m + 4 M) \cos[\theta_s[t]]) \sin[\theta_s[t]] \theta_s'[t]), \\
& \left. -\frac{1}{4} l^2 m (\cos[\theta_{ns}[t]] - 2 \cos[\theta_s[t]]) (\sin[\theta_{ns}[t]] \phi'[t] + \cos[\theta_{ns}[t]] \cos[\phi[t]] \omega'[t]), \right. \\
& \left. \frac{1}{4} l^2 (2 m \cos[\theta_{ns}[t]] - (5 m + 4 M) \cos[\theta_s[t]]) \right. \\
& \quad \left. (\sin[\theta_s[t]] \phi'[t] + \cos[\theta_s[t]] \cos[\phi[t]] \omega'[t]) \right\}, \\
& \left\{ -\frac{1}{4} l^2 m (\cos[\theta_{ns}[t]] \cos[\phi[t]] \right. \\
& \quad (- (\cos[\theta_{ns}[t]] - 2 \cos[\theta_s[t]]) \phi'[t] + \cos[\phi[t]] (\sin[\theta_{ns}[t]] - 2 \sin[\theta_s[t]]) \omega'[t]) + \\
& \quad 2 \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] (\theta_s'[t] + \sin[\phi[t]] \omega'[t])), \\
& \left. \frac{1}{4} l^2 m (\cos[\theta_{ns}[t]] - 2 \cos[\theta_s[t]]) (\sin[\theta_{ns}[t]] \phi'[t] + \cos[\theta_{ns}[t]] \cos[\phi[t]] \omega'[t]), \right. \\
& \left. 0, \right. \\
& \left. -\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] (\theta_s'[t] + \sin[\phi[t]] \omega'[t]) \right\}, \\
& \left\{ \frac{1}{8} l^2 (4 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t] + 2 \cos[\theta_s[t]] \right. \\
& \quad (-2 m \cos[\theta_{ns}[t]] + (5 m + 4 M) \cos[\theta_s[t])) \cos[\phi[t]] \phi'[t] + (4 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - \\
& \quad (5 m + 4 M) \cos[\phi[t]]^2 \sin[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] \sin[\phi[t]]^2) \omega'[t]), \\
& \left. \frac{1}{4} l^2 (-2 m \cos[\theta_{ns}[t]] + (5 m + 4 M) \cos[\theta_s[t]]) (\sin[\theta_s[t]] \phi'[t] + \cos[\theta_s[t]] \cos[\phi[t]] \omega'[t]), \right. \\
& \left. \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] (\theta_{ns}'[t] + \sin[\phi[t]] \omega'[t]), \right. \\
& \left. 0 \right\}
\end{aligned}$$

■ Potential Torques Vector

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g1 = D[PE, ω[t]]; g2 = D[PE, φ[t]]; g3 = D[PE, θns[t]]; g4 = D[PE, θs[t]];
gVect = {{g1}, {g2}, {g3}, {g4}} // FullSimplify

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$$\left\{ \{0\}, \left\{ \frac{1}{2} g l (m \cos[\theta_{ns}[t]] - (3m + 2M) \cos[\theta_s[t]]) \sin[\phi[t]] \right\}, \right. \\ \left. \left\{ \frac{1}{2} g l m \cos[\phi[t]] \sin[\theta_{ns}[t]] \right\}, \left\{ -\frac{1}{2} g l (3m + 2M) \cos[\phi[t]] \sin[\theta_s[t]] \right\} \right\}$$

■ Equations of Motion

$$\mathbf{B4x4} = \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 1, 1\}\}; \\ \mathbf{B2x2} = \{\{-1, 0\}, \{1, 1\}\};$$

$$\mathbf{Mmatrix.D}[\mathbf{qdot}, t] + \mathbf{Cmatrix.qdot} + \mathbf{gVect} = \mathbf{B4x4}.\{\{u1\}, \{u2\}, \{u3\}, \{u4\}\}$$

■ Impact Guard

The guard is the zero-level set of the height function:

$$\text{height} == 0 \\ 1 (\cos[\omega[t]] (-\sin[\theta_{ns}[t]] + \sin[\theta_s[t]]) \tan[\gamma] - \\ (\cos[\theta_{ns}[t]] - \cos[\theta_s[t]]) (\cos[\phi[t]] + \sin[\phi[t]] \sin[\omega[t]] \tan[\gamma])) == 0$$

and the negative region of the holonomic constraint's trajectory:

$$\text{holonomicTraj} < 0 \\ 1 (-\cos[\theta_{ns}[t]] \cos[\omega[t]] \tan[\gamma] + \sin[\theta_{ns}[t]] (\cos[\phi[t]] + \sin[\phi[t]] \sin[\omega[t]] \tan[\gamma])) \\ \theta_{ns}'[t] + \\ 1 (\cos[\theta_s[t]] \cos[\omega[t]] \tan[\gamma] - \sin[\theta_s[t]] (\cos[\phi[t]] + \sin[\phi[t]] \sin[\omega[t]] \tan[\gamma])) \\ \theta_s'[t] + 1 (\cos[\theta_{ns}[t]] - \cos[\theta_s[t]]) (\sin[\phi[t]] - \cos[\phi[t]] \sin[\omega[t]] \tan[\gamma]) \phi'[t] + \\ 1 ((-\cos[\theta_{ns}[t]] + \cos[\theta_s[t]]) \cos[\omega[t]] \sin[\phi[t]] + (\sin[\theta_{ns}[t]] - \sin[\theta_s[t]]) \sin[\omega[t]]) \\ \tan[\gamma] \omega'[t] < 0$$

Reduction-Based Control

■ Passivity-Based control law

v_{θ}

$$\left\{ \left\{ p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4g (-m \cos[\beta - \theta_{ns}[t]] + (3m + 2M) \cos[\beta - \theta_s[t]]) + \right. \right. \right. \\ \left. \left. \left. l (m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5m + 4M) \theta_s'[t]^2) \right) \right\}, \right. \\ \left. \left\{ -p (\theta_{ns}'[t] + \theta_s'[t]) \left(-E2Dref + \frac{1}{8} l (4g (-m \cos[\beta - \theta_{ns}[t]] + (3m + 2M) \cos[\beta - \theta_s[t]]) + \right. \right. \right. \\ \left. \left. \left. l (m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5m + 4M) \theta_s'[t]^2) \right) \right\} \right\}$$

`vtheta = FullSimplify[Inverse[B2x2]].`

`(Take[gVect, -2] - (Take[gVect, -2] /. {Θns[t] → Θns[t] - Beta, Θs[t] → Θs[t] - Beta}) /.
{ω[t] → 0, φ[t] → 0}) + vthetatilde // Simplify`

$$\left\{ \left\{ -\frac{1}{2} g l m \sin[\text{Beta} - \Theta_{ns}[t]] - \frac{1}{2} g l m \sin[\Theta_{ns}[t]] + \right. \right.$$

$$\left. p \Theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l \left(4 g (-m \cos[\text{Beta} - \Theta_{ns}[t]] + (3 m + 2 M) \cos[\text{Beta} - \Theta_s[t]]) + \right. \right.$$

$$\left. \left. l m \Theta_{ns}'[t]^2 - 4 l m \cos[\Theta_{ns}[t] - \Theta_s[t]] \Theta_{ns}'[t] \Theta_s'[t] + l (5 m + 4 M) \Theta_s'[t]^2 \right) \right\},$$

$$\left\{ \frac{1}{2} \left(g l m \sin[\text{Beta} - \Theta_{ns}[t]] + g l m \sin[\Theta_{ns}[t]] - g l (3 m + 2 M) \sin[\text{Beta} - \Theta_s[t]] - \right. \right.$$

$$\left. g l (3 m + 2 M) \sin[\Theta_s[t]] - \right.$$

$$\left. \left. 2 p (\Theta_{ns}'[t] + \Theta_s'[t]) \left(-E2Dref + \frac{1}{8} l \left(4 g (-m \cos[\text{Beta} - \Theta_{ns}[t]] + (3 m + 2 M) \cos[\text{Beta} - \Theta_s[t]]) + \right. \right. \right. \right.$$

$$\left. \left. \left. l m \Theta_{ns}'[t]^2 - 4 l m \cos[\Theta_{ns}[t] - \Theta_s[t]] \Theta_{ns}'[t] \Theta_s'[t] + l (5 m + 4 M) \Theta_s'[t]^2 \right) \right) \right\} \right\}$$

■ Zero Dynamics control law

`v12`

$$\left\{ \left\{ -16 \left(-r - \frac{1}{16} Lp (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + \right. \right. \right.$$

$$\left. \left. l^2 (14 m + 4 M + m \cos[2 \Theta_{ns}[t]] + (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + \right. \right.$$

$$\left. \left. l (l m \cos[2 \Theta_{ns}[t]] + l (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 l m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) \right) + \right.$$

$$\left. \left. 8 l^2 (3 m + 2 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]]^2 \right) + \frac{1}{16} \right.$$

$$\left. \left(-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \Theta_{ns}[t]] + 8 m \cos[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + l (-2 l m \sin[2 \Theta_{ns}[t]] + \right. \right.$$

$$\left. \left. 8 l m \cos[\Theta_{ns}[t]] \sin[\Theta_s[t]]) \right) + 16 l^2 m \sin[\Theta_{ns}[t] - \Theta_s[t]] \sin[\phi[t]]^2 \Theta_{ns}'[t] + \right.$$

$$\left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\Theta_s[t]] \sin[\Theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \Theta_s[t]]) + \right.$$

$$\left. \left. l (8 l m \cos[\Theta_s[t]] \sin[\Theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \Theta_s[t]]) \right) - \right.$$

$$\left. \left. 16 l^2 m \sin[\Theta_{ns}[t] - \Theta_s[t]] \sin[\phi[t]]^2 \Theta_s'[t] + \right. \right.$$

$$\left. \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right.$$

$$\left. \left. 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \Theta_{ns}[t]] + \right. \right.$$

$$\left. \left. (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + l (l m \cos[2 \Theta_{ns}[t]] + \right. \right.$$

$$\left. \left. l (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 l m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) \right) \sin[\phi[t]] \phi'[t] \right\}$$

$$\left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] \Theta_{ns}'[t] - \right.$$

$$\left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] \Theta_s'[t] + \right.$$

$$\left. \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2 \Theta_{ns}[t]] + (5 m + 4 M) \sin[2 \Theta_s[t]] - 4 m \sin[\Theta_{ns}[t] + \Theta_s[t]]) \phi'[t] - \right.$$

$$\left. \frac{1}{16} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + \right.$$

$$\left. \left. l^2 (14 m + 4 M + m \cos[2 \Theta_{ns}[t]] + (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + \right. \right.$$

$$\begin{aligned}
& \left. \left(1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \right. \\
& \left. \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \omega'[t] \right) / \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right\}, \\
& \left\{ 4096 \left(\frac{1}{128} c (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 \\
& \quad \left. \left. (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t]) \\
& \left(-c + \frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{8} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \theta_s'[t] \right) - \\
& \quad \frac{1}{131072} K_p (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right)^2 \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \phi'[t] + \\
& \quad \frac{1}{32} (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right) \\
& \left(-\frac{1}{8} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \\
& \left(-\frac{1}{8} l^2 l_p \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) + \right. \\
& \quad \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_{ns}'[t] + \\
& \quad \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 \right. \\
& \quad \left. (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \Big) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) + \\
& \frac{1}{16} \left(16 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad 2 \cos[\phi[t]] \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad \quad (5m + 4M) \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + l (1m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad \quad \left. \left. 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \sin[\phi[t]] \right) + \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \frac{1}{4} l^2 \right. \\
& \quad (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right)^2 - \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad (5m + 4M) \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. \left. 1 (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \right) + \\
& \quad 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left(2 v \phi 3D + 2 c Kp (-d + \phi[t]) + \right. \\
& \quad \left. 2 c \phi'[t] - \frac{1}{16} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \theta_{ns}'[t] \phi'[t] - \right. \\
& \quad \frac{1}{4} l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5m + 4M) \sin[2 \theta_s[t]]) \theta_s'[t] \phi'[t] - \\
& \quad \frac{1}{4} l^2 Lp \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \quad \left. \omega'[t] - \frac{1}{2} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \right. \\
& \quad \frac{1}{4} l^2 (2m \cos[2 \theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \\
& \quad \frac{1}{2} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \\
& \quad \frac{1}{4} l^2 (2 (5m + 4M) \cos[2 \theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \\
& \quad \left. \frac{1}{16} \left(16 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \right. \\
& \quad \quad \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \cos[2 \theta_s[t]] + \right. \\
& \quad \quad \quad \left. \left. 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + l (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + \right. \right. \\
& \quad \quad \quad \left. \left. \left. \left. 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) \right) \sin[\phi[t]] \omega'[t]^2 \right) \right) \right) \Big/ \\
& \left(\left(4 l^2 (13m + 6M) + l (-4 l (7m + 2M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l (-8m \cos[\theta_{ns}[t]] \right. \right. \\
& \quad \left. \left. \cos[\theta_s[t]] + (5m + 4M) \cos[2 \theta_s[t]]) \right) \right) \\
& \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \right. \right. \\
& \quad \left. \left. \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1 (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \right) + \\
& \quad \left. \left. 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \right) \Big\}
\end{aligned}$$

Total Reduction-Based Control Law

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Min[Max[First[utotal[[1]]], -sat1], sat1]
```

$$\begin{aligned}
& \text{Min} \left[\text{sat1}, \text{Max} \left[-\text{sat1}, \left(16 r \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_{ns}'[t] + \right. \right. \right. \\
& \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_s'[t] - \\
& \quad \left. \left. \left. \frac{1}{8} l^2 \text{Cos}[\phi[t]] (m \text{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_s[t]] - 4 m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) \right] / \\
& \quad \left(-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + \right. \\
& \quad \quad (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \\
& \quad \quad \left. 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) - \\
& \quad \left(16 \left(-r - \frac{1}{16} \text{Lp} (-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \quad (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \\
& \quad \quad \left. 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \\
& \quad \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) + \frac{1}{16} \\
& \quad \quad \left(-\text{Cos}[\phi[t]]^2 (l^2 (-2 m \text{Sin}[2 \theta_{ns}[t]] + 8 m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + 1 (-2 l m \text{Sin}[2 \theta_{ns}[t]] + \right. \\
& \quad \quad \left. 8 l m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + 16 l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]]^2 \theta_{ns}'[t] + \\
& \quad \quad \frac{1}{16} (-\text{Cos}[\phi[t]]^2 (l^2 (8 m \text{Cos}[\theta_s[t]] \text{Sin}[\theta_{ns}[t]] - 2 (5 m + 4 M) \text{Sin}[2 \theta_s[t]]) + \\
& \quad \quad \left. 1 (8 l m \text{Cos}[\theta_s[t]] \text{Sin}[\theta_{ns}[t]] - 2 l (5 m + 4 M) \text{Sin}[2 \theta_s[t])) \right) - \\
& \quad \quad \left. 16 l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]]^2 \right) \theta_s'[t] + \\
& \quad \quad \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] \text{Sin}[\phi[t]] + \\
& \quad \quad \left. 2 \text{Cos}[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + \right. \\
& \quad \quad \quad (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + \\
& \quad \quad \quad \left. 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) \text{Sin}[\phi[t]] \phi'[t] \left. \right) \\
& \quad \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_{ns}'[t] - \right. \\
& \quad \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_s'[t] + \\
& \quad \quad \left. \frac{1}{8} l^2 \text{Cos}[\phi[t]] (m \text{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_s[t]] - 4 m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] - \right. \\
& \quad \quad \left. \frac{1}{16} (-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + \right. \\
& \quad \quad \quad l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) + \\
& \quad \quad \quad \left. 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \\
& \quad \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \omega'[t] \left. \right) \left. \right) / \\
& \quad \left(-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + \right. \\
& \quad \quad (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \\
& \quad \quad \left. 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \left. \right) \left. \right) \left. \right)
\end{aligned}$$

Min[Max[First[utotal[[2]]], -sat2], sat2]

$$\begin{aligned}
& \text{Min} \left[\text{sat2}, \text{Max} \left[-\text{sat2}, -\frac{1}{2} g l (-m \text{Cos}[\theta_{ns}[t]] + (3m + 2M) \text{Cos}[\theta_s[t]]) \text{Sin}[\phi[t]] - \right. \right. \\
& \quad \frac{1}{32} \left(16 l^2 (3m + 2M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] \text{Sin}[\phi[t]] + 2 \text{Cos}[\phi[t]] (-2 l^2 (13m + 6M) + \right. \\
& \quad \quad \left. \left. l^2 (14m + 4M + m \text{Cos}[2\theta_{ns}[t]] + (5m + 4M) \text{Cos}[2\theta_s[t]] + 8m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \right. \right. \\
& \quad \quad \left. \left. l (1m \text{Cos}[2\theta_{ns}[t]] + 1 (5m + 4M) \text{Cos}[2\theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) \text{Sin}[\phi[t]] \right) \\
& \quad \omega'[t]^2 + \theta_{ns}'[t] \left(\frac{1}{32} l (32 l m \text{Cos}[\theta_s[t]] \text{Sin}[\theta_{ns}[t]] - 8 l m \text{Sin}[2\theta_{ns}[t]]) \phi'[t] + \right. \\
& \quad \left(\frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] - \right. \\
& \quad \quad \left. \frac{1}{8} l^2 (2m \text{Cos}[2\theta_{ns}[t]] - 4m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \text{Cos}[\phi[t]] \right) \omega'[t] \left. \right) + \\
& \quad \theta_s'[t] \left(\frac{1}{8} l^2 (8m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]] - 2 (5m + 4M) \text{Sin}[2\theta_s[t]]) \phi'[t] + \right. \\
& \quad \left(-\frac{1}{4} l^2 (5m + 4M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] - \right. \\
& \quad \quad \left. \frac{1}{8} l^2 (2 (5m + 4M) \text{Cos}[2\theta_s[t]] - 4m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \text{Cos}[\phi[t]] \right) \omega'[t] \left. \right) + \\
& \quad \left(524 288 \text{Cos}[\phi[t]] (m \text{Sin}[2\theta_{ns}[t]] + (5m + 4M) \text{Sin}[2\theta_s[t]] - 4m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left(\frac{1}{2048} l^6 m (-3m - 4M + 2m \text{Cos}[2(\theta_{ns}[t] - \theta_s[t])]) (4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \right. \\
& \quad \quad \left. \left. \text{Cos}[2\theta_{ns}[t]] + 4 l (-8m \text{Cos}[\theta_{ns}[t]] \text{Cos}[\theta_s[t]] + (5m + 4M) \text{Cos}[2\theta_s[t]])) \right) \right. \\
& \quad \quad \left. \text{Cos}[\phi[t]]^2 (m \text{Sin}[2\theta_{ns}[t]] + (5m + 4M) \text{Sin}[2\theta_s[t]] - 4m \text{Sin}[\theta_{ns}[t] + \theta_s[t]])^2 - \right. \\
& \quad \quad \frac{1}{256} (4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \text{Cos}[2\theta_{ns}[t]] + \\
& \quad \quad \quad \left. 4 l (-8m \text{Cos}[\theta_{ns}[t]] \text{Cos}[\theta_s[t]] + (5m + 4M) \text{Cos}[2\theta_s[t]])) \right)^2 \\
& \quad \quad \left(\frac{1}{16} l^4 m^2 (5m + 4M) (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]])^2 \text{Sin}[\phi[t]]^2 - \frac{1}{4} l^4 m^2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \quad \quad \left. (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) (5m + 4M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 + \right. \\
& \quad \quad \quad \left. \frac{1}{16} l^4 m (5m + 4M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]])^2 \text{Sin}[\phi[t]]^2 \right) \left. \right) \left. \right) \\
& \quad \left(-r \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_{ns}'[t] + \right. \right. \\
& \quad \quad \frac{1}{4} l^2 (5m + 4M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \text{Cos}[\phi[t]] \\
& \quad \quad \left. \left. (m \text{Sin}[2\theta_{ns}[t]] + (5m + 4M) \text{Sin}[2\theta_s[t]] - 4m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) + \\
& \quad \frac{1}{16} \left(-\text{Cos}[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \text{Cos}[2\theta_{ns}[t]] + (5m + 4M) \text{Cos}[2\theta_s[t]] + \right. \\
& \quad \quad \left. 8m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + 1 (1m \text{Cos}[2\theta_{ns}[t]] + 1 (5m + 4M) \text{Cos}[2\theta_s[t]] + 8 l m \right. \\
& \quad \quad \left. \left. \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) + 8 l^2 (3m + 2M - 2m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \right. \\
& \quad \left(\frac{1}{2} l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]] \right. \\
& \quad \quad \left. \theta_s'[t]^2 + \frac{1}{8} l^2 (m \text{Sin}[2\theta_{ns}[t]] + (5m + 4M) \text{Sin}[2\theta_s[t]] - 4m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \right)
\end{aligned}$$

$$\begin{aligned}
& \sin[\phi[t]] \phi'[t]^2 + \frac{1}{16} \left(16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad 2 \cos[\phi[t]] \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \cos[2 \theta_s[t]] + \right. \\
& \quad \quad 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + \\
& \quad \quad \quad 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \left. \right) \phi'[t] \omega'[t] + \\
& \left(16 \left(-r - \frac{1}{16} \text{Lp} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \right. \right. \right. \\
& \quad \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[\\
& \quad \quad \quad 2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left. \right) + \frac{1}{16} \left(-\cos[\phi[t]]^2 \right. \\
& \quad \left(l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (-2 l m \sin[2 \theta_{ns}[t]] + \right. \\
& \quad \quad \left. 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \left. \right) \\
& \theta_{ns}'[t] + \frac{1}{16} \left(-\cos[\phi[t]]^2 \left(l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \right. \right. \\
& \quad \sin[2 \theta_s[t]]) + 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \\
& \quad \quad \left. \sin[2 \theta_s[t])) \right) - 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \left. \right) \theta_s'[t] + \\
& \frac{1}{16} \left(16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad 2 \cos[\phi[t]] \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \\
& \quad \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + \\
& \quad \quad \quad 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \left. \right) \phi'[t] \left. \right) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \\
& \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left(m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] - \right. \\
& \quad \frac{1}{16} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad \quad 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \omega'[t] \right) \left. \right) / \\
& \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) + \\
& \theta_s'[t] \left(\left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \right. \\
& \quad \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] + \\
& \quad \frac{1}{16} \left(-\cos[\phi[t]]^2 \left(l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]] \right) + \right. \\
& \quad \quad \left. 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) \right) - \\
& \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \omega'[t] \left. \right) +
\end{aligned}$$

$$\begin{aligned}
& \Theta_{ns}'[t] \left(\left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\Theta_{ns}[t] - \Theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. (2 m \cos[2 \Theta_{ns}[t]] - 4 m \cos[\Theta_{ns}[t] + \Theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] + \right. \\
& \quad \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \Theta_{ns}[t]] + 8 m \cos[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + \right. \\
& \quad \left. 1 (-2 l m \sin[2 \Theta_{ns}[t]] + 8 l m \cos[\Theta_{ns}[t]] \sin[\Theta_s[t])) \right) + \\
& \quad \left. 16 l^2 m \sin[\Theta_{ns}[t] - \Theta_s[t]] \sin[\phi[t]]^2 \right) \omega'[t] \Big) \Big) / \\
& \left(m (-3 m - 4 M + 2 m \cos[2 (\Theta_{ns}[t] - \Theta_s[t])]) (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + \right. \\
& \quad \left. 4 l m \cos[2 \Theta_{ns}[t]] + 4 l (-8 m \cos[\Theta_{ns}[t]] \cos[\Theta_s[t]] + (5 m + 4 M) \cos[2 \Theta_s[t]])) \right)^2 \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \Theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \cos[2 \Theta_s[t]] + 8 m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + \\
& \quad 1 (l m \cos[2 \Theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 l m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t])) \Big) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]]^2 \right)^3 \Big) - \\
& \left(8 \cos[\phi[t]] (m \sin[2 \Theta_{ns}[t]] + (5 m + 4 M) \sin[2 \Theta_s[t]] - 4 m \sin[\Theta_{ns}[t] + \Theta_s[t]]) \right. \\
& \quad \left(-\frac{1}{2} l^2 m \cos[\Theta_{ns}[t] - \Theta_s[t]] \right. \\
& \quad \left. (-1 + 2 \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] \right) \\
& \quad \left(p \Theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4 g (-m \cos[\text{Beta} - \Theta_{ns}[t]] + (3 m + 2 M) \cos[\text{Beta} - \Theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1 (m \Theta_{ns}'[t]^2 - 4 m \cos[\Theta_{ns}[t] - \Theta_s[t]] \Theta_{ns}'[t] \Theta_s'[t] + (5 m + 4 M) \Theta_s'[t]^2) \right) \right) - \\
& \quad p (\Theta_{ns}'[t] + \Theta_s'[t]) \left(-E2Dref + \frac{1}{8} l (4 g (-m \cos[\text{Beta} - \Theta_{ns}[t]] + (3 m + 2 M) \cos[\text{Beta} - \Theta_s[t]]) + \right. \\
& \quad \left. \left. 1 (m \Theta_{ns}'[t]^2 - 4 m \cos[\Theta_{ns}[t] - \Theta_s[t]] \Theta_{ns}'[t] \Theta_s'[t] + (5 m + 4 M) \Theta_s'[t]^2) \right) \right) - \\
& \quad \left(65536 \left(\frac{1}{1024} c^2 l^2 (8 m \cos[\Theta_{ns}[t]] \sin[\Theta_s[t]] - 2 (5 m + 4 M) \sin[2 \Theta_s[t]]) \right. \right. \\
& \quad \left. \left. (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \Theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \cos[2 \Theta_s[t]] + 8 m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t]]) + 1 (l m \cos[2 \Theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 1 (5 m + 4 M) \cos[2 \Theta_s[t]] + 8 l m \sin[\Theta_{ns}[t]] \sin[\Theta_s[t])) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t])^2 + \\
& \quad \frac{1}{1024} (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \Theta_{ns}[t]] + 4 l \\
& \quad (-8 m \cos[\Theta_{ns}[t]] \cos[\Theta_s[t]] + (5 m + 4 M) \cos[2 \Theta_s[t])) \Big)^2 \\
& \quad \left(2 \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] \Theta_{ns}'[t] - \right. \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\Theta_{ns}[t] - \Theta_s[t]]) \sin[\phi[t]] \Theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. \left. (m \sin[2 \Theta_{ns}[t]] + (5 m + 4 M) \sin[2 \Theta_s[t]] - 4 m \sin[\Theta_{ns}[t] + \Theta_s[t]]) \phi'[t] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{16} r (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) (-s + \omega[t]) + \right. \\
& \quad \frac{1}{64} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] (-\cos[\phi[t]]^2 \\
& \quad (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \\
& \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta ns'[t] + \frac{1}{128} l^2 \cos[\phi[t]] \right. \\
& \quad (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \\
& \quad \left. (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \phi'[t] + \frac{1}{4} l^2 (5 m + \right. \\
& \quad \left. 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right. \\
& \quad \left(\frac{1}{8} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta ns[t]] + 8 m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad \left. l (-2 l m \sin[2 \theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t])) + \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta ns'[t] + \frac{1}{16} \right. \\
& \quad \left. (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta s'[t] + \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \right. \\
& \quad \left. \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] (-2 l^2 (13 m + \right. \\
& \quad \left. 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \right. \\
& \quad \left. \sin[\theta ns[t]] \sin[\theta s[t]]) + l (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[\right. \\
& \quad \left. 2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \sin[\phi[t]] \phi'[t] \right) \left. \right) + \\
& \quad \frac{1}{256} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& \quad (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad l (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \\
& \quad \sin[\theta s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2)^2 \\
& \quad \left(2 g l (3 m + 2 M) \sin[\text{Beta} - \theta s[t]] + 2 l^2 m \sin[\theta ns[t] - \theta s[t]] \theta ns'[t]^2 - \right. \\
& \quad \frac{1}{4} l^2 (8 m \cos[\theta ns[t]] \sin[\theta s[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) \phi'[t]^2 + \\
& \quad 4 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] \omega'[t] + \\
& \quad \left. 4 \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. (2 (5 m + 4 M) \cos[2 \theta s[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] - \right. \\
& \quad \left. \frac{1}{8} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{16384} l^2 m (-3m - 4M + 2m \cos[2(\theta_{ns}[t] - \theta_s[t])]) \\
& \left(4l^2 (13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right)^2 \\
& \left(-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 \right. \\
& \quad \left. (1m \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \\
& \left(\left(4096 \left(\frac{1}{128} c (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \right. \right. \\
& \quad \left. \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \right. \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t]) \\
& \left(-c + \frac{1}{32} l (32lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8lm \sin[2\theta_{ns}[t]]) \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{8} l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) \theta_s'[t] \right) - \\
& \quad \frac{1}{131072} Kp \left(4l^2 (13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right)^2 \\
& \left(-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \phi'[t] + \\
& \quad \frac{1}{32} \left(4l^2 (13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l \right. \\
& \quad \left. (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right) \\
& \left(-\frac{1}{8} (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left(-\frac{1}{8} l^2 Lp \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) + \right. \\
& \quad \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \frac{1}{8} l^2 (2m \cos[2\theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \\
& \quad \theta_{ns}'[t] + \left(\frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \frac{1}{8} l^2 (2(5m + 4M) \cos[2\theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \\
& \left(r(-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \left(m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]] \right) \phi'[t] \Big) + \\
& \frac{1}{16} \left(16 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad 2 \cos[\phi[t]] \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \quad \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) \sin[\phi[t]] \Big) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. \left(m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]] \right) \phi'[t] \right)^2 - \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \right. \\
& \quad \quad \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + 1 \\
& \quad \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + 8 l^2 (3m + 2M - \\
& \quad \quad 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \Big)^2 \left(2c K_p (-d + \phi[t]) + 2c \phi'[t] - \right. \\
& \quad \frac{1}{16} l (32lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8lm \sin[2\theta_{ns}[t]]) \theta_{ns}'[t] \phi'[t] - \frac{1}{4} \\
& \quad l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) \theta_s'[t] \phi'[t] - \frac{1}{4} l^2 L_p \\
& \quad \cos[\phi[t]] (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \quad \omega'[t] - \frac{1}{2} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \\
& \quad \frac{1}{4} l^2 (2m \cos[2\theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \\
& \quad \frac{1}{2} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \\
& \quad \frac{1}{4} l^2 (2(5m + 4M) \cos[2\theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \\
& \quad \left. \frac{1}{16} \left(16 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \right. \\
& \quad \quad \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + \right. \\
& \quad \quad \left. 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \right. \\
& \quad \quad \left. \left. \left. \left. \left. \left. \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) \sin[\phi[t]] \right) \omega'[t]^2 \right) \right) \right) \right) \Big) \Big) \Big) \Big) \Big) / \\
& \left(\left(4 l^2 (13m + 6M) + 1 (-4l (7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l (-8m \cos[\theta_{ns}[t]] \right. \right. \\
& \quad \quad \left. \left. \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right) \right) \\
& \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \right. \\
& \quad \quad \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad \left. 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \Big) -
\end{aligned}$$

$$\begin{aligned}
& \left(4096 \left(-\frac{1}{32} (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right) \right. \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) \\
& \left(-\frac{1}{4} l^2 \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left(\frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \quad \left. 1 (-2 l m \sin[2 \theta_{ns}[t]] + 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t]])) + \right. \\
& \quad \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \theta_{ns}'[t] + \frac{1}{16} \right. \\
& \quad \quad \left. (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) + \right. \\
& \quad \quad \left. 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t]])) - \right. \\
& \quad \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \theta_s'[t] \right) + \\
& \quad \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (l m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]]) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \\
& \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) + \\
& \quad \frac{1}{256} (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \\
& \quad \quad (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad 1 (l m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \\
& \quad \left(\left(\left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \right. \right. \\
& \quad \quad \left. \left. (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \right) \\
& \quad \theta_{ns}'[t] + \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \\
& \quad \quad \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] \right) - \frac{1}{8} l^2 \cos[\phi[t]] \\
& (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]] \theta_s'[t]^2 - \frac{1}{4} l^2 (2m \cos[2\theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right. \\
& \quad \left. \theta_{ns}'[t] \phi'[t] - \frac{1}{4} l^2 (2(5m + 4M) \cos[2\theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \cos[\phi[t]] \theta_s'[t] \phi'[t] + \frac{1}{8} l^2 (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - \right. \\
& \quad \left. 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\phi[t]] \phi'[t]^2 + r \omega'[t] \right) \Bigg) + \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]]) + \right. \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + \\
& \quad \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& 8l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left(-2c^2(-d + \phi[t]) + \right. \\
& \frac{1}{32} \left(4l^2(13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right) \\
& \left(-\frac{1}{16} \left(16l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \right. \\
& \quad \left. \left(-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + \right. \right. \\
& \quad \left. \left. 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + 1(5m + 4M) \right. \right. \\
& \quad \left. \left. \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \right) \omega'[t]^2 + \right. \\
& 2\theta_{ns}'[t] \left(\frac{1}{32} l(32lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8lm \sin[2\theta_{ns}[t]]) \right. \\
& \quad \left. \phi'[t] + \left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2m \cos[2\theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \omega'[t] \right) + \\
& 2\theta_s'[t] \left(\frac{1}{8} l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) \phi'[t] + \right. \\
& \quad \left(-\frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2(5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \omega'[t] \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(\left(4l^2(13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l(-8m \cos[\theta_{ns}[t]] \right. \right. \\
& \quad \left. \left. \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right) \right. \\
& \left(-\cos[\phi[t]]^2 (-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \right. \\
& \left. 1(1m \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) +
\end{aligned}$$

$$\begin{aligned}
& 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \left(-r (-s + \omega[t]) - \right. \\
& \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \left. \phi'[t] \right) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \\
& \left(\frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2m \sin[2\theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]])) + \right. \\
& \quad \left. l (-2lm \sin[2\theta_{ns}[t]] + 8lm \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_{ns}'[t] + \frac{1}{8} \\
& (-\cos[\phi[t]]^2 (l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) + \\
& \quad \left. l (8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t])) \right) - \\
& 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_s'[t] + \frac{1}{8} (16 l^2 (3m + 2M - 2m \\
& \quad \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] (-2 l^2 (13m + \\
& \quad 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + 8m \\
& \quad \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + l (lm \cos[2\theta_{ns}[t]] + l(5m + 4M) \cos[\\
& \quad \quad 2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \phi'[t] \left. \right) + \\
& \frac{1}{4} (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad \left. l (lm \cos[2\theta_{ns}[t]] + l(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \right. \\
& \quad \left. \sin[\theta_s[t])) \right) + 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \\
& \sin[\phi[t]]^2 \left(\left(-l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t] + \right. \right. \\
& \left. \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 (2m \cos[2\theta_{ns}[t]] - \right. \right. \\
& \quad \left. \left. 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) \left(r (-s + \omega[t]) + \frac{1}{4} l^2 \right. \\
& (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) - \\
& \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t]^2 - \right. \\
& \quad \left. \frac{1}{2} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \phi'[t] + \right. \\
& \quad \left. \frac{1}{8} l^2 (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\right. \\
& \quad \left. \phi[t] \right) \phi'[t]^2 + \theta_s'[t] \left(-l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t] + \left(\frac{1}{4} l^2 \right. \right. \\
& \quad \left. \left. (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2(5m + 4M) \cos[\right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(262144 \left(-\frac{1}{8192} 1^8 m^2 (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (-3m - 4M + 2m \cos[2(\theta_{ns}[t] - \theta_s[t])]) \right) \right. \\
& \quad \left(4l^2 (13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + \right. \\
& \quad \quad \left. 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t]])) \right) \cos[\phi[t]]^2 \\
& \quad \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]])^2 \sin[\phi[t]] + \right. \\
& \quad \left. \frac{1}{1024} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (4l^2 (13m + 6M) + 1(-4l(7m + 2M) + \right. \\
& \quad \quad \left. 4lm \cos[2\theta_{ns}[t]] + 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right)^2 \\
& \quad \left. \sin[\phi[t]] \left(\frac{1}{16} l^4 m^2 (5m + 4M) (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 - \frac{1}{4} l^4 \right. \right. \\
& \quad \quad \left. m^2 \cos[\theta_{ns}[t] - \theta_s[t]] (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \right. \\
& \quad \quad \left. \left. \sin[\phi[t]]^2 + \frac{1}{16} l^4 m (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 \right) \right) \\
& \left(-\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 + \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t]^2 + \right. \\
& \quad \left(16r \left(r(-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \right. \\
& \quad \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \quad \left. \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) / \\
& \quad \left(-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + \right. \\
& \quad \quad \left. 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(lm \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \right. \\
& \quad \quad \left. \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) - \\
& \quad \phi'[t] \left(\frac{1}{8} l^2 (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \quad \left. \sin[\phi[t]] \phi'[t] + \frac{1}{16} (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad \quad \left. 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + 1(lm \cos[2\theta_{ns}[t]] + \right. \\
& \quad \quad \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \omega'[t] \right) - \\
& \quad \left(16 \left(-r - \frac{1}{16} Lp (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \right. \\
& \quad \quad \left. 1(lm \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) + \frac{1}{16} (-\cos[\phi[t]]^2 \\
& \quad \quad \left(l^2 (-2m \sin[2\theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] + 1(-2lm \sin[2\theta_{ns}[t]] + \right. \\
& \quad \quad \left. 8lm \cos[\theta_{ns}[t]] \sin[\theta_s[t])) + 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \\
& \quad \quad \theta_{ns}'[t] + \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t])) + \\
& \quad \quad \left. 1(8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t])) \right) - \\
& \quad \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_s'[t] + \\
& \quad \quad \frac{1}{16} (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad \quad \left. 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t]] + 1 (1 m \cos[2 \theta n s[t]] + \\
& 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t]]) \sin[\phi[t]] \phi'[t] \right) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \theta n s'[t] - \right. \\
& \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \theta s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& (m \sin[2 \theta n s[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta n s[t] + \theta s[t]]) \phi'[t] - \\
& \frac{1}{16} (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + \\
& (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t]]) + \\
& 1 (1 m \cos[2 \theta n s[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) + \\
& \left. \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]]^2 \right) \omega'[t] \right) \Bigg) / \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + (5 m + 4 M) \right. \\
& \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t]]) + \\
& 1 (1 m \cos[2 \theta n s[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) + \\
& \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]]^2 \right) - \\
& \theta s'[t] \left(\left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \cos[\phi[t]] - \right. \right. \\
& \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta s[t]] - 4 m \cos[\theta n s[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] + \right. \\
& \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta n s[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \left. 1 (8 l m \cos[\theta s[t]] \sin[\theta n s[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) \right) - \\
& \left. 16 l^2 m \sin[\theta n s[t] - \theta s[t]] \sin[\phi[t]]^2 \right) \omega'[t] \Bigg) - \\
& \theta n s'[t] \left(\left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2 m \cos[2 \theta n s[t]] - \right. \right. \\
& \left. \left. 4 m \cos[\theta n s[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] + \right. \\
& \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta n s[t]] + 8 m \cos[\theta n s[t]] \sin[\theta s[t]]) + \right. \\
& \left. 1 (-2 l m \sin[2 \theta n s[t]] + 8 l m \cos[\theta n s[t]] \sin[\theta s[t])) \right) + \\
& \left. 16 l^2 m \sin[\theta n s[t] - \theta s[t]] \sin[\phi[t]]^2 \right) \omega'[t] \Bigg) \Bigg) / \\
& \left(l^2 m (-3 m - 4 M + 2 m \cos[2 (\theta n s[t] - \theta s[t])]) (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + \right. \\
& \left. 4 l m \cos[2 \theta n s[t]] + 4 l (-8 m \cos[\theta n s[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t])) \right) \Bigg)^2 \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + (5 m + 4 M) \right. \\
& \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t]]) + \\
& 1 (1 m \cos[2 \theta n s[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) + \\
& \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]]^2 \right) \Bigg) + \\
& \left(16 m (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \left(-\frac{1}{2} l^2 m \cos[\theta n s[t] - \theta s[t]] \right. \right. \\
& \left. \left. (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] + \right. \right. \\
& \left. \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l \left(4g (-m \cos[\text{Beta} - \theta_{ns}[t]] + (3m + 2M) \cos[\text{Beta} - \theta_s[t]]) \right) + \right. \right. \\
& \quad \left. \left. l \left(m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5m + 4M) \theta_s'[t]^2 \right) \right) \right) - \\
& p \left(\theta_{ns}'[t] + \theta_s'[t] \right) \left(-E2Dref + \frac{1}{8} l \left(4g (-m \cos[\text{Beta} - \theta_{ns}[t]] + (3m + 2M) \cos[\text{Beta} - \theta_s[t]]) \right) + \right. \\
& \quad \left. \left. l \left(m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5m + 4M) \theta_s'[t]^2 \right) \right) \right) - \\
& \left(65536 \left(\frac{1}{1024} c^2 l^2 \left(8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2(5m + 4M) \sin[2\theta_s[t]] \right) \right. \right. \\
& \quad \left. \left(-\cos[\phi[t]]^2 \left(-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \right. \right. \\
& \quad \quad \left. \left. \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) + l(1m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \quad \left. \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) \right) + \\
& \quad \left. 8l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) (-d + \phi[t])^2 + \\
& \quad \frac{1}{1024} \left(4l^2(13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l \right. \\
& \quad \quad \left. \left. (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t]]) \right) \right)^2 \\
& \left(2 \left(-r(-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \right. \\
& \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) \\
& \left(-\frac{1}{16} r (-\cos[\phi[t]]^2 \left(l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) \right) + \right. \\
& \quad \left. 1(8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t]]) \right) - \\
& \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) (-s + \omega[t]) + \\
& \quad \frac{1}{64} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \left(-\cos[\phi[t]]^2 \right. \\
& \quad \left. \left(l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1(8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t]]) \right) \right) - \\
& \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_{ns}'[t] + \frac{1}{128} l^2 \cos[\phi[t]] \\
& \quad \left(m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]] \right) \\
& \quad \left(-\cos[\phi[t]]^2 \left(l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1(8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t]]) \right) \right) - \\
& \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \phi'[t] + \frac{1}{4} l^2 (5m + \\
& \quad \left. 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right) \\
& \left(\frac{1}{8} \left(-\cos[\phi[t]]^2 \left(l^2 (-2m \sin[2\theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \right. \right. \\
& \quad \left. \left. 1(-2lm \sin[2\theta_{ns}[t]] + 8lm \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \right) + \\
& \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_{ns}'[t] + \frac{1}{16} \\
& \quad \left(-\cos[\phi[t]]^2 \left(l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1(8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t]]) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_s'[t] + \frac{1}{8} \left(16 l^2 (3 m + 2 M - 2 m \right. \\
& \quad \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \left(-2 l^2 (13 m + \right. \\
& \quad \quad 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \\
& \quad \quad \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[\\
& \quad \quad \quad 2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \sin[\phi[t]] \phi'[t] \left. \right) + \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \\
& \quad \quad \sin[\theta_s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left. \right)^2 \\
& \left(2 g l (3 m + 2 M) \sin[\beta - \theta_s[t]] + 2 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t]^2 - \right. \\
& \quad \frac{1}{4} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \phi'[t]^2 + \\
& \quad 4 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t] \omega'[t] + \\
& \quad \left. 4 \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 \right. \right. \\
& \quad \quad \left. \left. (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] - \right. \\
& \quad \frac{1}{8} \left(-\cos[\phi[t]]^2 \left(l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) + \right. \right. \\
& \quad \quad \left. \left. 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t]]) \right) - \right. \\
& \quad \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \omega'[t]^2 \left. \right) + \\
& \frac{1}{4} \left(-\cos[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + 8 l^2 \\
& \quad \quad (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left(\left(-l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \right. \\
& \quad \quad \sin[\phi[t]] \theta_{ns}'[t] + \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} \right. \\
& \quad \quad \left. \left. l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \\
& \quad \quad \theta_s[t]]) \phi'[t] \left. \right) + \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \\
& \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \sin[\phi[t]] \theta_s'[t]^2 + \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \\
& \quad \quad \left. \left. (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_{ns}'[t] \phi'[t] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{8} l^2 (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \sin[\phi[t]] \phi'[t]^2 + 2 \theta_s'[t] \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \phi'[t] \right) + r \omega'[t] \Big) \Big) \Big) \Big) \Big) \Big) / \\
& \left((4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l (-8m \cos[\theta_{ns}[t]] \right. \\
& \left. \cos[\theta_s[t]] + (5m + 4M) \cos[2 \theta_s[t]])) \right)^2 \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \\
& \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& 1 (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \Big) \Big) \Big) / \\
& ((-3m - 4M + 2m \cos[2 (\theta_{ns}[t] - \theta_s[t])]) (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + \\
& l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& 1 (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) + \\
& \left(16 l^4 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\right. \\
& \left. \phi[\right. \\
& \left. t] \right. \\
& (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - \\
& 4 \\
& m \\
& \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\phi[t]] \\
& \left. \left(\left(4096 \left(\frac{1}{128} c (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + \right. \right. \right. \right. \right. \\
& \left. \left. \left. (5m + 4M) \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] + 1 (1m \cos[2 \theta_{ns}[t]] + \right. \right. \right. \right. \\
& \left. \left. \left. 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \right. \right. \right. \\
& \left. \left. 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) (-d + \phi[t]) \right. \\
& \left. \left(-c + \frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \theta_{ns}'[t] + \right. \right. \\
& \left. \left. \frac{1}{8} l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5m + 4M) \sin[2 \theta_s[t]]) \theta_s'[t] \right) - \right. \\
& \left. \frac{1}{131072} Kp (4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \\
& \left. 4 l (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2 \theta_s[t])) \right)^2 \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \\
& \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] + 1 (1m \cos[2 \theta_{ns}[t]] + \\
& 1 (5m + 4M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \phi'[t] + \\
& \frac{1}{32} (4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \\
& (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2 \theta_s[t])) \\
& \left. \left(-\frac{1}{8} (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \text{Cos}[2 \theta s[t]] + 8 m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t]] + 1 (1 m \text{Cos}[2 \theta ns[t]] + \\
& 1 (5 m + 4 M) \text{Cos}[2 \theta s[t]] + 8 l m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t])) + \\
& 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]]^2 \left(-\frac{1}{8} l^2 \text{Lp} \text{Cos}[\phi[t]] \right. \\
& (m \text{Sin}[2 \theta ns[t]] + (5 m + 4 M) \text{Sin}[2 \theta s[t]] - 4 m \text{Sin}[\theta ns[t] + \theta s[t])) + \\
& \left. \left(-\frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Cos}[\phi[t]] + \right. \right. \\
& \left. \left. \frac{1}{8} l^2 (2 m \text{Cos}[2 \theta ns[t]] - 4 m \text{Cos}[\theta ns[t] + \theta s[t]]) \text{Cos}[\phi[t]] \right) \right) \\
& \theta ns'[t] + \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Cos}[\phi[t]] + \right. \\
& \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \text{Cos}[2 \theta s[t]] - 4 m \text{Cos}[\theta ns[t] + \theta s[t]]) \text{Cos}[\phi[t]] \right) \theta s'[t] \Big) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]] \theta ns'[t] + \right. \\
& \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \text{Cos}[\phi[t]] \right. \\
& \left. (m \text{Sin}[2 \theta ns[t]] + (5 m + 4 M) \text{Sin}[2 \theta s[t]] - 4 m \text{Sin}[\theta ns[t] + \theta s[t]]) \phi'[t] \right) + \\
& \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Cos}[\phi[t]] \text{Sin}[\phi[t]] + \\
& 2 \text{Cos}[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta ns[t]] + (5 m + 4 M) \\
& \text{Cos}[2 \theta s[t]] + 8 m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t]] + 1 (1 m \text{Cos}[2 \theta ns[t]] + \\
& 1 (5 m + 4 M) \text{Cos}[2 \theta s[t]] + 8 l m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t])) \text{Sin}[\phi[t]]) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]] \theta ns'[t] + \right. \\
& \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \text{Cos}[\phi[t]] \right. \\
& \left. (m \text{Sin}[2 \theta ns[t]] + (5 m + 4 M) \text{Sin}[2 \theta s[t]] - 4 m \text{Sin}[\theta ns[t] + \theta s[t]]) \phi'[t] \right)^2 - \\
& \frac{1}{256} (-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta ns[t]] + (5 m + 4 M) \\
& \text{Cos}[2 \theta s[t]] + 8 m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t]] + 1 (1 m \text{Cos}[2 \theta ns[t]] + 1 \\
& (5 m + 4 M) \text{Cos}[2 \theta s[t]] + 8 l m \text{Sin}[\theta ns[t]] \text{Sin}[\theta s[t])) + 8 l^2 (3 m + 2 M - \\
& 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Sin}[\phi[t]]^2 \Big)^2 \left(2 c \text{Kp} (-d + \phi[t]) + 2 c \phi'[t] - \right. \\
& \left. \frac{1}{16} l (32 l m \text{Cos}[\theta s[t]] \text{Sin}[\theta ns[t]] - 8 l m \text{Sin}[2 \theta ns[t]]) \theta ns'[t] \phi'[t] - \frac{1}{4} \right. \\
& \left. l^2 (8 m \text{Cos}[\theta ns[t]] \text{Sin}[\theta s[t]] - 2 (5 m + 4 M) \text{Sin}[2 \theta s[t]]) \theta s'[t] \phi'[t] - \frac{1}{4} l^2 \text{Lp} \right. \\
& \left. \text{Cos}[\phi[t]] (m \text{Sin}[2 \theta ns[t]] + (5 m + 4 M) \text{Sin}[2 \theta s[t]] - 4 m \text{Sin}[\theta ns[t] + \theta s[t])) \right) \\
& \omega'[t] - \frac{1}{2} l^2 m (-1 + 2 \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Cos}[\phi[t]] \theta ns'[t] \omega'[t] + \\
& \frac{1}{4} l^2 (2 m \text{Cos}[2 \theta ns[t]] - 4 m \text{Cos}[\theta ns[t] + \theta s[t]]) \text{Cos}[\phi[t]] \theta ns'[t] \omega'[t] + \\
& \frac{1}{2} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta ns[t] - \theta s[t]]) \text{Cos}[\phi[t]] \theta s'[t] \omega'[t] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} l^2 (2 (5 m + 4 M) \cos[2 \theta s[t]] - 4 m \cos[\theta n s[t] + \theta s[t]]) \cos[\phi[t]] \theta s'[t] \omega'[t] + \\
& \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \\
& \quad (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + (5 m + 4 M) \cos[2 \theta s[t]] + \\
& \quad \quad 8 m \sin[\theta n s[t]] \sin[\theta s[t]]) + l (1 m \cos[2 \theta n s[t]] + l (5 m + 4 M) \\
& \quad \quad \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) \sin[\phi[t]] \omega'[t]^2) \Big) \Big) \Big) / \\
& \left((4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta n s[t]] + 4 l (-8 m \cos[\theta n s[t]] \right. \\
& \quad \left. \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]])) \right) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + (5 m + 4 M) \\
& \quad \quad \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t])) + \\
& \quad \quad 1 (1 m \cos[2 \theta n s[t]] + l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) + \\
& \quad \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t])) \sin[\phi[t]]^2) - \\
& \quad \left(4096 \left(-\frac{1}{32} (4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta n s[t]] + \right. \right. \\
& \quad \quad \left. \left. 4 l (-8 m \cos[\theta n s[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]])) \right) \right) \\
& \quad \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \theta n s'[t] + \right. \\
& \quad \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta n s[t] - \theta s[t])) \sin[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \quad \left. (m \sin[2 \theta n s[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta n s[t] + \theta s[t]]) \phi'[t] \right) \\
& \quad \left(-\frac{1}{4} l^2 \cos[\phi[t]] (m \sin[2 \theta n s[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta n s[t] + \theta s[t]]) \right) \\
& \quad \left(\frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta n s[t]] + 8 m \cos[\theta n s[t]] \sin[\theta s[t]]) + \right. \\
& \quad \quad \left. 1 (-2 l m \sin[2 \theta n s[t]] + 8 l m \cos[\theta n s[t]] \sin[\theta s[t])) \right) + \\
& \quad \quad 16 l^2 m \sin[\theta n s[t] - \theta s[t]] \sin[\phi[t]]^2) \theta n s'[t] + \frac{1}{16} \\
& \quad \quad (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta n s[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]] + \\
& \quad \quad \quad 1 (8 l m \cos[\theta s[t]] \sin[\theta n s[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \\
& \quad \quad \quad 16 l^2 m \sin[\theta n s[t] - \theta s[t]] \sin[\phi[t]]^2) \theta s'[t] \Big) + \\
& \quad \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\theta n s[t] - \theta s[t])) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad \quad 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta n s[t]] + (5 m + 4 M) \\
& \quad \quad \quad \cos[2 \theta s[t]] + 8 m \sin[\theta n s[t]] \sin[\theta s[t]]) + l (1 m \cos[2 \theta n s[t]] + \\
& \quad \quad \quad 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta n s[t]] \sin[\theta s[t])) \sin[\phi[t]]) \\
& \quad \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta n s[t] - \theta s[t]]) \sin[\phi[t]] \theta n s'[t] - \right. \\
& \quad \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta n s[t] - \theta s[t])) \sin[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \quad \left. (m \sin[2 \theta n s[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta n s[t] + \theta s[t]]) \phi'[t] \right) \Big) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{256} \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \right. \\
& \quad \left. (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]]) \right) \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \\
& \left(\left(\left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \right. \right. \\
& \quad \left. \left. (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \right. \\
& \quad \left. \theta_{ns}'[t] + \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \right) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] \right) - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \quad \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]] \theta_s'[t]^2 - \frac{1}{4} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right. \\
& \quad \left. \theta_{ns}'[t] \phi'[t] - \frac{1}{4} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \cos[\phi[t]] \theta_s'[t] \phi'[t] + \frac{1}{8} l^2 (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - \right. \\
& \quad \left. 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\phi[t]] \phi'[t]^2 + r \omega'[t] \right) \Big) + \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \left(-2 c^2 (-d + \phi[t]) + \right. \\
& \quad \left. \frac{1}{32} \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]]) \right) \right) \\
& \quad \left(-\frac{1}{16} \left(16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \right. \\
& \quad \left. \left. (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \cos[2 \theta_s[t]] + \right. \right. \\
& \quad \left. \left. 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \sin[\phi[t]] \right) \omega'[t]^2 + \\
& \quad \left. 2 \theta_{ns}'[t] \left(\frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \phi'[t] + \left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \\
& \quad \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \omega'[t] + \\
& 2 \theta_s'[t] \left(\frac{1}{8} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \phi'[t] + \right. \\
& \quad \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2 (5 m + 4 M) \right. \\
& \quad \quad \left. \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \omega'[t] \left. \right) \left. \right) \left. \right) \left. \right) / \\
& \left((4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l (-8 m \cos[\theta_{ns}[t]] \right. \\
& \quad \left. \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \left. \right) \left. \right) \left. \right) / \\
& \left((4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \\
& \quad \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \left. \right) - \left(16 \left(-l^2 \right. \right. \\
& \quad m \\
& \quad (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \\
& \quad \sin[\\
& \quad \quad \phi[t]] \\
& \quad \left(-\frac{1}{4} l^2 m (5 m + 4 M) (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] + \right. \\
& \quad \quad \frac{1}{2} l^2 m \cos[\theta_{ns}[t] - \theta_s[t]] \\
& \quad \quad \left. (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right) + \\
& \quad \frac{1}{16} l^2 m (-3 m - 4 M + 2 m \cos[2 (\theta_{ns}[t] - \theta_s[t])]) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + \\
& \quad \quad l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \left. \right) \\
& \left(-p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4 g (-m \cos[\text{Beta} - \theta_{ns}[t]] + (3 m + 2 M) \cos[\text{Beta} - \theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1 (m \theta_{ns}'[t]^2 - 4 m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5 m + 4 M) \theta_s'[t]^2) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(65536 \left(\frac{1}{4096} c^2 l (32 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 8 l m \sin[2 \theta ns[t]]) \right. \right. \\
& \quad \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \right. \\
& \quad \quad \left. \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + l (l m \cos[2 \theta ns[t]] + \right. \\
& \quad \quad \left. 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t])^2 + \\
& \quad \frac{1}{1024} \left(4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta ns[t]] + 4 l \right. \\
& \quad \quad \left. (-8 m \cos[\theta ns[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]]) \right)^2 \\
& \quad \left(-2 \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta ns'[t] + \right. \right. \\
& \quad \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \quad \left. \left. (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \phi'[t] \right) \right) \\
& \quad \left(\frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta ns[t]] + 8 m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad \quad \left. 1 (-2 l m \sin[2 \theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t])) \right) + \\
& \quad \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \left(-r (-s + \omega[t]) - \right. \\
& \quad \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \quad \left. \left. (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \right. \right. \\
& \quad \quad \left. \left. \phi'[t] \right) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right) \\
& \quad \left(\frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta ns[t]] + 8 m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad \quad \left. 1 (-2 l m \sin[2 \theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t])) \right) + \\
& \quad \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \theta ns'[t] + \frac{1}{8} \\
& \quad \quad \left(-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \quad \left. 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) \right) - \\
& \quad \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \theta s'[t] + \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \\
& \quad \quad \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] (-2 l^2 (13 m + \\
& \quad \quad 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \\
& \quad \quad \sin[\theta ns[t]] \sin[\theta s[t]]) + l (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[\\
& \quad \quad \quad 2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \sin[\phi[t]] \phi'[t] \right) \right) + \\
& \quad \frac{1}{4} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& \quad \quad (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad \quad 1 (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \\
& \quad \quad \sin[\theta s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \\
& \quad \quad \sin[\phi[t]]^2 \left(\left(-l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta s'[t] + \right. \right.
\end{aligned}$$

$$\begin{aligned} & (l^2 m (-3 m - 4 M + 2 m \cos[2 (\theta_{ns}[t] - \theta_s[t])]) \\ & (-\cos[\phi[t]]^2 \\ & (-2 l^2 (13 m + 6 M) + \\ & l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\ & l (1 m \cos[2 \theta_{ns}[t]] + l (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\ & 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \\ & \sin[\phi[t]]^2)) \end{aligned}$$

Min[Max[First[utotal[[4]], -sat4], sat4]

$$\begin{aligned} & \text{Min}[\text{sat4}, \text{Max}[-\text{sat4}, \frac{1}{2} \left(g l m \cos[\phi[t]] \sin[\theta_{ns}[t]] - g l (3 m + 2 M) \cos[\phi[t]] \sin[\theta_s[t]] + \right. \\ & l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t]^2 - l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \theta_s'[t]^2 - \\ & \frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \phi'[t]^2 - \\ & \frac{1}{8} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \phi'[t]^2 + \\ & 2 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t] \omega'[t] - 2 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \\ & \left. \sin[\phi[t]] \theta_s'[t] \omega'[t] + 2 \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \right. \\ & \left. \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] + \right. \\ & 2 \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\ & \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] - \\ & \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t])) + \\ & l (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) - \\ & 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \omega'[t]^2 - \\ & \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + l (-2 l m \sin[2 \theta_{ns}[t]] + \\ & 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t])) + 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \omega'[t]^2 + \\ & \left(524 288 \left(-\frac{1}{8192} l^8 m^2 (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (-3 m - 4 M + 2 m \cos[2 (\theta_{ns}[t] - \theta_s[t])) \right) \right. \\ & (4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \\ & 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \cos[\phi[t]]^2 \\ & (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]])^2 \sin[\phi[t]] + \\ & \frac{1}{1024} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \\ & \left. \cos[2 \theta_{ns}[t]] + 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \right)^2 \\ & \left. \sin[\phi[t]] \left(\frac{1}{16} l^4 m^2 (5 m + 4 M) (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 - \frac{1}{4} l^4 \right. \right. \\ & \left. \left. m^2 \cos[\theta_{ns}[t] - \theta_s[t]] (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \right. \right. \\ & \left. \left. \sin[\phi[t]]^2 + \frac{1}{16} l^4 m (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 + \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]] \theta_s'[t]^2 + \left(16 r \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right. \right. \right. \\
& \quad \left. \left. \theta_{ns}'[t] + \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \right. \\
& \quad \left. \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) / \\
& \quad \left(-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \left. 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) - \\
& \quad \phi'[t] \left(\frac{1}{8} l^2 (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \sin[\phi[t]] \phi'[t] + \frac{1}{16} (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad \left. 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \omega'[t] \right) - \\
& \quad \left(16 \left(-r - \frac{1}{16} Lp (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \right. \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) + \frac{1}{16} \\
& \quad \left(-\cos[\phi[t]]^2 (l^2 (-2m \sin[2\theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \left. 1 (-2lm \sin[2\theta_{ns}[t]] + 8lm \cos[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_{ns}'[t] + \frac{1}{16} (-\cos[\phi[t]]^2 \right. \\
& \quad \left. (l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]] + 1(8lm \cos[\theta_s[t]] \right. \\
& \quad \left. \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t])) - 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]]^2 \theta_s'[t] + \frac{1}{16} (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \right. \\
& \quad \left. \sin[\phi[t]] + 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \phi'[t] \right) \\
& \quad \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \frac{1}{4} \right. \\
& \quad \left. l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \phi'[t] - \frac{1}{16} (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) +
\end{aligned}$$

$$\begin{aligned}
& 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \omega'[t] \Big) \Big) \Big) / \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) - \\
\theta_s'[t] & \left(\left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] + \right. \\
& \quad \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) + \right. \\
& \quad \quad \left. 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \omega'[t] \right) - \\
\theta_{ns}'[t] & \left(\left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] + \right. \\
& \quad \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \quad \left. 1 (-2 l m \sin[2 \theta_{ns}[t]] + 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t])) + \right. \\
& \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \omega'[t] \right) \Big) \Big) \Big) / \\
& (l^2 m (-3 m - 4 M + 2 m \cos[2 (\theta_{ns}[t] - \theta_s[t])]) (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 \\
& \quad 1 m \cos[2 \theta_{ns}[t]] + 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \Big)^2 \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 \\
& \quad (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \Big)^2 + \\
& \left(524 288 \left(\frac{1}{8192} l^8 m (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) (-3 m - 4 M + 2 m \cos[2 (\theta_{ns}[t] - \theta_s[t])]) \right. \right. \\
& \quad \left. \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \quad \left. \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \cos[\phi[t]]^2 \right. \right. \\
& \quad \left. \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]])^2 \sin[\phi[t]] - \right. \right. \\
& \quad \left. \left. \frac{1}{1024} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l \right. \right. \\
& \quad \quad \left. \left. m \cos[2 \theta_{ns}[t]] + 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \right) \right) \Big)^2 \\
& \sin[\phi[t]] \left(\frac{1}{16} l^4 m^2 (5 m + 4 M) (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 - \frac{1}{4} l^4 \right. \\
& \quad m^2 \cos[\theta_{ns}[t] - \theta_s[t]] (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \\
& \quad \left. \sin[\phi[t]]^2 + \frac{1}{16} l^4 m (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]])^2 \sin[\phi[t]]^2 \right) \Big) \\
& \left(-\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 + \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \right. \\
& \quad \left. \sin[\phi[t]] \theta_s'[t]^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(16 r \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \frac{1}{4} \right. \right. \\
& \quad l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) / \\
& \left(-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \left. \right) + \\
& \quad 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left. \right) - \\
& \phi'[t] \left(\frac{1}{8} l^2 (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \sin[\phi[t]] \phi'[t] + \frac{1}{16} (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \right) \omega'[t] \left. \right) - \\
& \left(16 \left(-r - \frac{1}{16} Lp (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \left. \right) + \frac{1}{16} \\
& \quad (-\cos[\phi[t]]^2 (l^2 (-2m \sin[2\theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (-2lm \sin[2\theta_{ns}[t]] + 8lm \cos[\theta_{ns}[t]] \sin[\theta_s[t])) \left. \right) + \\
& \quad 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_{ns}'[t] + \frac{1}{16} \\
& \quad (-\cos[\phi[t]]^2 (l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) + \\
& \quad 1 (8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2\theta_s[t])) \left. \right) - \\
& \quad 16l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_s'[t] + \frac{1}{16} \\
& \quad (16l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \right) \phi'[t] \left. \right) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \frac{1}{4} \right. \\
& \quad l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \quad \left. \phi'[t] - \frac{1}{16} (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \left. 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \right) + \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \omega'[t] \left. \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Cos}[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. l (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) - \\
& \theta s'[t] \left(\left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] - \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] - 4 m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \text{Cos}[\phi[t]] \right) \phi'[t] + \right. \\
& \quad \left. \frac{1}{16} \left(-\text{Cos}[\phi[t]]^2 \left(l^2 (8 m \text{Cos}[\theta_s[t]] \text{Sin}[\theta_{ns}[t]] - 2 (5 m + 4 M) \text{Sin}[2 \theta_s[t]]) + \right. \right. \right. \\
& \quad \left. \left. \left. l (8 l m \text{Cos}[\theta_s[t]] \text{Sin}[\theta_{ns}[t]] - 2 l (5 m + 4 M) \text{Sin}[2 \theta_s[t]]) \right) - \right. \right. \\
& \quad \left. \left. 16 l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]]^2 \right) \omega'[t] \right) - \\
& \theta_{ns}'[t] \left(\left(-\frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] - \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. (2 m \text{Cos}[2 \theta_{ns}[t]] - 4 m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \text{Cos}[\phi[t]] \right) \phi'[t] + \right. \\
& \quad \left. \frac{1}{16} \left(-\text{Cos}[\phi[t]]^2 \left(l^2 (-2 m \text{Sin}[2 \theta_{ns}[t]] + 8 m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \right. \right. \right. \\
& \quad \left. \left. \left. l (-2 l m \text{Sin}[2 \theta_{ns}[t]] + 8 l m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) \right) + \right. \right. \\
& \quad \left. \left. 16 l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]]^2 \right) \omega'[t] \right) \right) / \\
& \left(l^2 m (-3 m - 4 M + 2 m \text{Cos}[2 (\theta_{ns}[t] - \theta_s[t])]) \left(4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 \right. \right. \\
& \quad \left. \left. l m \text{Cos}[2 \theta_{ns}[t]] + 4 l (-8 m \text{Cos}[\theta_{ns}[t]] \text{Cos}[\theta_s[t]] + (5 m + 4 M) \text{Cos}[2 \theta_s[t]]) \right) \right)^2 \\
& \left(-\text{Cos}[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + l \right. \right. \\
& \quad \left. \left. (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right)^2 \Big) - \\
& \left(32 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \left(-\frac{1}{2} l^2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]] \right. \right. \\
& \quad \left. \left. (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] + \right. \right. \\
& \quad \left. \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \right) \right) \\
& \left(p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4 g (-m \text{Cos}[\text{Beta} - \theta_{ns}[t]] + (3 m + 2 M) \text{Cos}[\text{Beta} - \theta_s[t]]) + \right. \right. \\
& \quad \left. \left. l (m \theta_{ns}'[t]^2 - 4 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5 m + 4 M) \theta_s'[t]^2) \right) \right) - p (\theta_{ns}'[\\
& \quad t] + \theta_s'[t]) \left(-E2Dref + \frac{1}{8} l (4 g (-m \text{Cos}[\text{Beta} - \theta_{ns}[t]] + (3 m + 2 M) \text{Cos}[\text{Beta} - \theta_s[t]]) + \right. \\
& \quad \left. \left. l (m \theta_{ns}'[t]^2 - 4 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5 m + 4 M) \theta_s'[t]^2) \right) \right) - \\
& \left(65536 \left(\frac{1}{1024} c^2 l^2 (8 m \text{Cos}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]] - 2 (5 m + 4 M) \text{Sin}[2 \theta_s[t]]) \right. \right. \\
& \quad \left. \left. (-\text{Cos}[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \right. \\
& \quad \left. \left. \left. \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + l (1 m \text{Cos}[2 \theta_{ns}[t]] + \right. \right. \right. \\
& \quad \left. \left. \left. 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 (-d + \phi[t])^2 + \\
& \frac{1}{1024} \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \right. \\
& \quad \left. (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right)^2 \\
& \left(2 \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \\
& \left(-\frac{1}{16} r (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \right. \\
& \quad \sin[2 \theta_s[t]] + 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \\
& \quad \sin[2 \theta_s[t]])) - 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \\
& \quad (-s + \omega[t]) + \frac{1}{64} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \\
& \quad (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]] + \\
& \quad 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) - \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \theta_{ns}'[t] + \frac{1}{128} l^2 \cos[\phi[t]] \\
& \quad (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \quad (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]] + \\
& \quad 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) - \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \phi'[t] + \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right) \\
& \left(\frac{1}{8} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad 1 (-2 l m \sin[2 \theta_{ns}[t]] + 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t]])) + \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \theta_{ns}'[t] + \frac{1}{16} (-\cos[\phi[t]]^2 \\
& \quad (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]] + \\
& \quad 1 (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) - \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2) \theta_s'[t] + \\
& \quad \left. \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad 1 (l m \cos[2 \theta_{ns}[t]] + l (5 m + 4 M) \cos[2 \theta_s[t]] + \\
& \quad \left. 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \phi'[t] \right) \Big) + \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad 1 (l m \cos[2 \theta_{ns}[t]] + l (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \\
& \quad \left. \sin[\theta_s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2
\end{aligned}$$

$$\begin{aligned}
& \left(2 g l (3 m + 2 M) \sin[\text{Beta} - \theta s[t]] + 2 l^2 m \sin[\theta ns[t] - \theta s[t]] \theta ns'[t]^2 - \right. \\
& \quad \frac{1}{4} l^2 (8 m \cos[\theta ns[t]] \sin[\theta s[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) \phi'[t]^2 + \\
& \quad 4 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] \omega'[t] + \\
& \quad 4 \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 (2 (5 m + 4 M) \right. \\
& \quad \quad \left. \cos[2 \theta s[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] - \\
& \quad \frac{1}{8} \left(-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2 \theta s[t]]) + \right. \\
& \quad \quad \left. l (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t])) - \right. \\
& \quad \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \right) \omega'[t]^2 \Bigg) + \frac{1}{4} \\
& \quad \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \right. \\
& \quad \quad \left. \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + l (l m \cos[2 \theta ns[t]] + \right. \\
& \quad \quad \left. l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \right) + 8 l^2 \\
& \quad \left(3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]] \right) \sin[\phi[t]]^2 \Bigg) \left(\left(-l^2 m \sin[\theta ns[t] - \theta s[t]] \right. \right. \\
& \quad \left. \left. \sin[\phi[t]] \theta ns'[t] + \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] - \right. \right. \right. \\
& \quad \quad \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta s[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \right. \\
& \quad \left. \phi'[t] \right) \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right. \\
& \quad \left. \theta ns'[t] + \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \right. \\
& \quad \quad \left. \sin[\theta ns[t] + \theta s[t]]) \phi'[t] \right) + \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \\
& \quad \left. \sin[\phi[t]] \left(\frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t]^2 - \frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \right. \right. \\
& \quad \quad \left. \left. \sin[\phi[t]] \theta ns'[t]^2 + \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \right. \right. \right. \\
& \quad \quad \left. \left. \cos[\phi[t]] - \frac{1}{8} l^2 (2 m \cos[2 \theta ns[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \right. \right. \\
& \quad \quad \left. \left. \cos[\phi[t]] \right) \theta ns'[t] \phi'[t] + \frac{1}{8} l^2 (m \sin[2 \theta ns[t]] + (5 m + 4 M) \right. \\
& \quad \quad \left. \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \sin[\phi[t]] \phi'[t]^2 + \right. \\
& \quad \quad \left. 2 \theta s'[t] \left(\frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] + \frac{1}{4} l^2 \right. \right. \\
& \quad \quad \left. \left. (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \phi'[t] \right) + r \omega'[t] \right) \Bigg) \Bigg) \Bigg) \Bigg) \Bigg) / \\
& \left(\left(4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta ns[t]] + 4 l (-8 m \cos[\theta ns[t]] \right. \right. \\
& \quad \left. \left. \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t])) \right) \right)^2 \\
& \quad \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \right. \right. \\
& \quad \quad \left. \left. \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + l (l m \cos[2 \theta ns[t]] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) + \\
& \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \Bigg) / \\
& \left((-3m - 4M + 2m \cos[2(\theta_{ns}[t] - \theta_s[t])]) (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + \right. \\
& \quad 1^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) + \\
& \left(32 \left(1^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right. \right. \\
& \quad \left(-\frac{1}{2} l^2 m \cos[\theta_{ns}[t] - \theta_s[t]] (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] + \right. \\
& \quad \left. \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right) + \right. \\
& \quad \left. \frac{1}{16} l^2 (-3m - 4M + 2m \cos[2(\theta_{ns}[t] - \theta_s[t])]) (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 \right. \\
& \quad \quad (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \quad 1 (1m \cos[2\theta_{ns}[t]] + 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad \quad \left. \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \right) \\
& \left(p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4g (-m \cos[\text{Beta} - \theta_{ns}[t]] + (3m + 2M) \cos[\text{Beta} - \theta_s[t])) + \right. \right. \\
& \quad 1 (m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + \\
& \quad \quad \left. \left. (5m + 4M) \theta_s'[t]^2 \right) \right) - p (\theta_{ns}'[t] + \theta_s'[t]) \\
& \left(-E2Dref + \frac{1}{8} l (4g (-m \cos[\text{Beta} - \theta_{ns}[t]] + (3m + 2M) \cos[\text{Beta} - \theta_s[t])) + \right. \\
& \quad \left. \left. 1 (m \theta_{ns}'[t]^2 - 4m \cos[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5m + 4M) \theta_s'[t]^2 \right) \right) - \\
& \left(65536 \left(\frac{1}{1024} c^2 l^2 (8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2(5m + 4M) \sin[2\theta_s[t]]) \right. \right. \\
& \quad (-\cos[\phi[t]]^2 (-2l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \\
& \quad \quad \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + 1 (1m \cos[2\theta_{ns}[t]] + \\
& \quad \quad 1 (5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \\
& \quad \quad \left. \left. 8l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t])^2 + \right. \\
& \quad \left. \frac{1}{1024} (4l^2 (13m + 6M) + 1 (-4l (7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l \right. \\
& \quad \quad \left. (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right)^2 \\
& \quad \left(2 \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \right. \\
& \quad \quad \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \quad \left. \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \right) \\
& \left(-\frac{1}{16} r (-\cos[\phi[t]]^2 (l^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \right. \right. \\
& \quad \quad \left. \left. \sin[2\theta_s[t]] + 1 (8lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l (5m + 4M) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sin[2\theta s[t]]) - 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \\
& (-s + \omega[t]) + \frac{1}{64} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \\
& (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2\theta s[t]]) + \\
& \quad 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2\theta s[t]))) - \\
& \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta ns'[t] + \frac{1}{128} l^2 \cos[\phi[t]] \\
& (m \sin[2\theta ns[t]] + (5 m + 4 M) \sin[2\theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \\
& (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2\theta s[t]]) + \\
& \quad 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2\theta s[t]))) - \\
& \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \phi'[t] + \\
& \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \\
& \left(\frac{1}{8} (-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2\theta ns[t]] + 8 m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad 1 (-2 l m \sin[2\theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t]))) + \\
& \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta ns'[t] + \frac{1}{16} (-\cos[\phi[t]]^2 \\
& \quad (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2\theta s[t]]) + \\
& \quad 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2\theta s[t]))) - \\
& \quad 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta s'[t] + \\
& \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t])) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2\theta ns[t]] + \\
& \quad (5 m + 4 M) \cos[2\theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + \\
& \quad 1 (1 m \cos[2\theta ns[t]] + 1 (5 m + 4 M) \cos[2\theta s[t]] + \\
& \quad \left. 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \sin[\phi[t]] \phi'[t] \right) + \\
& \frac{1}{256} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2\theta ns[t]] + \\
& \quad (5 m + 4 M) \cos[2\theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + \\
& \quad 1 (1 m \cos[2\theta ns[t]] + 1 (5 m + 4 M) \cos[2\theta s[t]] + 8 l m \sin[\theta ns[t]] \\
& \quad \sin[\theta s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2) \\
& \left(2 g l (3 m + 2 M) \sin[\beta - \theta s[t]] + 2 l^2 m \sin[\theta ns[t] - \theta s[t]] \theta ns'[t]^2 - \right. \\
& \quad \frac{1}{4} l^2 (8 m \cos[\theta ns[t]] \sin[\theta s[t]] - 2 (5 m + 4 M) \sin[2\theta s[t]]) \phi'[t]^2 + \\
& \quad 4 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] \omega'[t] + \\
& \quad 4 \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] + \frac{1}{8} l^2 (2 (5 m + 4 M) \right. \\
& \quad \left. \cos[2\theta s[t]] - 4 m \cos[\theta ns[t] + \theta s[t])) \cos[\phi[t]] \right) \phi'[t] \omega'[t] - \\
& \quad \left. \frac{1}{8} (-\cos[\phi[t]]^2 (l^2 (8 m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 (5 m + 4 M) \sin[2\theta s[t]]) + \right. \\
& \quad \left. 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2\theta s[t]))) - \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \omega'[t]^2 \right) + \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Cos}[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]] \right) + l (1 m \text{Cos}[2 \theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) \right) + 8 l^2 \\
& \quad \left. (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \left(\left(-l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \right. \right. \\
& \quad \left. \left. \text{Sin}[\phi[t]] \theta_{ns}'[t] + \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] - 4 m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \text{Cos}[\phi[t]] \right) \right) \right. \\
& \quad \left. \phi'[t] \right) \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]] \right. \\
& \quad \left. \theta_{ns}'[t] + \frac{1}{8} l^2 \text{Cos}[\phi[t]] (m \text{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_s[t]] - 4 m \right. \\
& \quad \left. \left. \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) + \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \right. \\
& \quad \left. \text{Sin}[\phi[t]] \left(\frac{1}{2} l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \right. \right. \\
& \quad \left. \left. \text{Sin}[\phi[t]] \theta_{ns}'[t]^2 + \left(-\frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \right. \right. \right. \\
& \quad \left. \left. \left. \text{Cos}[\phi[t]] - \frac{1}{8} l^2 (2 m \text{Cos}[2 \theta_{ns}[t]] - 4 m \text{Cos}[\theta_{ns}[t] + \theta_s[t]]) \right. \right. \right. \\
& \quad \left. \left. \left. \text{Cos}[\phi[t]] \right) \theta_{ns}'[t] \phi'[t] + \frac{1}{8} l^2 (m \text{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \text{Sin}[2 \theta_s[t]] - 4 m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \text{Sin}[\phi[t]] \phi'[t]^2 + \right. \right. \\
& \quad \left. \left. 2 \theta_s'[t] \left(\frac{1}{2} l^2 m \text{Sin}[\theta_{ns}[t] - \theta_s[t]] \text{Sin}[\phi[t]] \theta_{ns}'[t] + \frac{1}{4} l^2 \right. \right. \right. \\
& \quad \left. \left. \left. (5 m + 4 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] \phi'[t] \right) \right) \right) \right) \right) \right) / \\
& \left((4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \text{Cos}[2 \theta_{ns}[t]] + 4 l (-8 m \text{Cos}[\theta_{ns}[t]] \right. \right. \\
& \quad \left. \left. \text{Cos}[\theta_s[t]] + (5 m + 4 M) \text{Cos}[2 \theta_s[t]])) \right)^2 \right. \\
& \quad \left. (-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \\
& \quad \left. \left. \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + l (1 m \text{Cos}[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \right. \\
& \quad \left. \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \right) \right) / \\
& \left(l^2 (-3 m - 4 M + 2 m \text{Cos}[2 (\theta_{ns}[t] - \theta_s[t])]) (-\text{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + \right. \right. \\
& \quad \left. \left. l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1 (1 m \text{Cos}[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \text{Cos}[2 \theta_s[t]] + 8 l m \text{Sin}[\theta_{ns}[t]] \text{Sin}[\theta_s[t])) \right) + \right. \\
& \quad \left. \left. 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Sin}[\phi[t]]^2 \right) \right) - \\
& \left(32 l^4 m (-1 + 2 \text{Cos}[\theta_{ns}[t] - \theta_s[t]]) \text{Cos}[\phi[t]] \right. \\
& \quad \left. \text{Sin}[\phi[t]] \right. \\
& \quad \left. (m \text{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_s[t]] - \right. \\
& \quad \left. 4 m \text{Sin}[\theta_{ns}[t] + \theta_s[t]]) \text{Sin}[\phi[t]] \right. \\
& \quad \left. \text{Sin}[\phi[t]] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(4096 \left(\frac{1}{128} c (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]]) + (5 m + 4 M) \right. \right. \right. \\
& \quad \left. \left. \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t]) \\
& \left(-c + \frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{8} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \theta_s'[t] \right) - \\
& \frac{1}{131072} Kp \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \right)^2 \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]]) + (5 m + 4 M) \right. \\
& \quad \left. \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \phi'[t] + \frac{1}{32} \\
& \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \right. \\
& \quad \left. (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t])) \right) \\
& \left(-\frac{1}{8} (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]]) + \right. \\
& \quad \left. (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \right. \\
& \quad \left. \sin[\theta_s[t]]) \right) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \Big) \\
& \left(-\frac{1}{8} l^2 Lp \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - \right. \\
& \quad \left. 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) + \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \right) \\
& \theta_{ns}'[t] + \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \Big) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) + \\
& \frac{1}{16} \left(16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \right. \\
& \quad \left. 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]]) + (5 m + 4 M) \right. \\
& \quad \left. \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) \sin[\phi[t]] \Big)
\end{aligned}$$

$$\begin{aligned}
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \frac{1}{4} l^2 \right. \\
& \quad \left. (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[\right. \\
& \quad \left. 2\theta_{ns}[t] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right)^2 - \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + \right. \\
& \quad (5m + 4M) \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1(1m \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \\
& \quad \left. \sin[\theta_s[t]]) + 8l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \\
& \left(2c Kp (-d + \phi[t]) + 2c \phi'[t] - \frac{1}{16} l(32lm \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - \right. \\
& \quad 8lm \sin[2\theta_{ns}[t]]) \theta_{ns}'[t] \phi'[t] - \frac{1}{4} l^2(8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - \\
& \quad 2(5m + 4M) \sin[2\theta_s[t]]) \theta_s'[t] \phi'[t] - \frac{1}{4} l^2 Lp \cos[\phi[t]] \\
& \quad \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \omega'[t] - \right. \\
& \quad \left. \frac{1}{2} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (2m \cos[2\theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_{ns}'[t] \omega'[t] + \right. \\
& \quad \left. \frac{1}{2} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \frac{1}{4} l^2 \right. \\
& \quad \left. (2(5m + 4M) \cos[2\theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \theta_s'[t] \omega'[t] + \right. \\
& \quad \left. \frac{1}{16} (16l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \\
& \quad \left. (-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \cos[2\theta_s[t]] + \right. \\
& \quad \left. 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + 1(1m \cos[2\theta_{ns}[t]] + 1(5m + 4M) \cos[\right. \\
& \quad \left. 2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) \sin[\phi[t]] \omega'[t]^2 \right) \Big) \Big) \Big) \Big) \Big) / \\
& \left((4l^2(13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + 4l(-8m \cos[\right. \\
& \quad \left. \theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t]])) \right) \\
& \left(-\cos[\phi[t]]^2 (-2l^2(13m + 6M) + l^2(14m + 4M + m \cos[2\theta_{ns}[t]] + (5m + 4M) \right. \\
& \quad \left. \cos[2\theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1(1m \cos[2\theta_{ns}[t]] + \right. \\
& \quad \left. 1(5m + 4M) \cos[2\theta_s[t]] + 8lm \sin[\theta_{ns}[t]] \sin[\theta_s[t])) + \right. \\
& \quad \left. 8l^2(3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 - \\
& \left(4096 \left(-\frac{1}{32} (4l^2(13m + 6M) + 1(-4l(7m + 2M) + 4lm \cos[2\theta_{ns}[t]] + \right. \right. \\
& \quad \left. \left. 4l(-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2\theta_s[t])) \right) \right) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \left(-\frac{1}{4} \right. \\
& \quad \left. l^2 \cos[\phi[t]] (m \sin[2\theta_{ns}[t]] + (5m + 4M) \sin[2\theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{16} \left(-\text{Cos}[\phi[t]]^2 \left(l^2 \left(-2 m \text{Sin}[2 \theta_{\text{ns}}[t]] + 8 m \text{Cos}[\theta_{\text{ns}}[t]] \text{Sin}[\theta_{\text{s}}[t]] \right) + \right. \right. \right. \\
& \quad \left. \left. \left. l \left(-2 l m \text{Sin}[2 \theta_{\text{ns}}[t]] + 8 l m \text{Cos}[\theta_{\text{ns}}[t]] \text{Sin}[\theta_{\text{s}}[t]] \right) \right) \right) + \right. \\
& \quad \left. 16 l^2 m \text{Sin}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]] \text{Sin}[\phi[t]]^2 \right) \theta_{\text{ns}}'[t] + \frac{1}{16} \\
& \quad \left(-\text{Cos}[\phi[t]]^2 \left(l^2 \left(8 m \text{Cos}[\theta_{\text{s}}[t]] \text{Sin}[\theta_{\text{ns}}[t]] - 2 (5 m + 4 M) \text{Sin}[2 \theta_{\text{s}}[t]] \right) + \right. \right. \\
& \quad \left. \left. l \left(8 l m \text{Cos}[\theta_{\text{s}}[t]] \text{Sin}[\theta_{\text{ns}}[t]] - 2 l (5 m + 4 M) \text{Sin}[2 \theta_{\text{s}}[t]] \right) \right) - \right. \\
& \quad \left. \left. 16 l^2 m \text{Sin}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]] \text{Sin}[\phi[t]]^2 \right) \theta_{\text{s}}'[t] \right) + \\
& \frac{1}{16} \left(16 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] \text{Sin}[\phi[t]] + \right. \\
& \quad 2 \text{Cos}[\phi[t]] \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{\text{ns}}[t]] + (5 m + 4 M) \right. \\
& \quad \quad \left. \text{Cos}[2 \theta_{\text{s}}[t]] + 8 m \text{Sin}[\theta_{\text{ns}}[t]] \text{Sin}[\theta_{\text{s}}[t]]) + l (1 m \text{Cos}[2 \theta_{\text{ns}}[t]] + \right. \\
& \quad \quad \left. \left. l (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]] + 8 l m \text{Sin}[\theta_{\text{ns}}[t]] \text{Sin}[\theta_{\text{s}}[t]]) \right) \text{Sin}[\phi[t]] \right) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Sin}[\phi[t]] \theta_{\text{ns}}'[t] - \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Sin}[\phi[t]] \theta_{\text{s}}'[t] - \frac{1}{8} l^2 \text{Cos}[\phi[t]] \right. \\
& \quad \left. \left. (m \text{Sin}[2 \theta_{\text{ns}}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_{\text{s}}[t]] - 4 m \text{Sin}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \phi'[t] \right) \right) + \\
& \frac{1}{256} \left(4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \text{Cos}[2 \theta_{\text{ns}}[t]] + 4 l \right. \\
& \quad \left. (-8 m \text{Cos}[\theta_{\text{ns}}[t]] \text{Cos}[\theta_{\text{s}}[t]] + (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]]) \right) \\
& \left(-\text{Cos}[\phi[t]]^2 \left(-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \text{Cos}[2 \theta_{\text{ns}}[t]] + \right. \right. \\
& \quad \left. \left. (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]] + 8 m \text{Sin}[\theta_{\text{ns}}[t]] \text{Sin}[\theta_{\text{s}}[t]]) + \right. \right. \\
& \quad \left. \left. l (1 m \text{Cos}[2 \theta_{\text{ns}}[t]] + l (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]] + 8 l m \text{Sin}[\theta_{\text{ns}}[t]] \right. \right. \\
& \quad \quad \left. \left. \text{Sin}[\theta_{\text{s}}[t]]) \right) + 8 l^2 (3 m + 2 M - 2 m \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Sin}[\phi[t]]^2 \right) \\
& \left(\left(\left(\frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] - \frac{1}{8} l^2 (2 m \text{Cos}[2 \theta_{\text{ns}}[t]] - \right. \right. \right. \\
& \quad \left. \left. \left. 4 m \text{Cos}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] \right) \theta_{\text{ns}}'[t] + \right. \right. \\
& \quad \left. \left(-\frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] - \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. \left. (2 (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]] - 4 m \text{Cos}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] \right) \theta_{\text{s}}'[t] \right) \right) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Sin}[\phi[t]] \theta_{\text{ns}}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \text{Cos}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]]) \text{Sin}[\phi[t]] \theta_{\text{s}}'[t] \right) - \frac{1}{8} l^2 \text{Cos}[\phi[t]] \\
& (m \text{Sin}[2 \theta_{\text{ns}}[t]] + (5 m + 4 M) \text{Sin}[2 \theta_{\text{s}}[t]] - 4 m \text{Sin}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \\
& \left(\frac{1}{2} l^2 m \text{Sin}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]] \text{Sin}[\phi[t]] \theta_{\text{ns}}'[t]^2 - \frac{1}{2} l^2 m \text{Sin}[\theta_{\text{ns}}[t] - \theta_{\text{s}}[t]] \text{Sin}[\phi[t]] \right. \\
& \quad \left. \theta_{\text{s}}'[t]^2 - \frac{1}{4} l^2 (2 m \text{Cos}[2 \theta_{\text{ns}}[t]] - 4 m \text{Cos}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \text{Cos}[\phi[t]] \right. \\
& \quad \left. \theta_{\text{ns}}'[t] \phi'[t] - \frac{1}{4} l^2 (2 (5 m + 4 M) \text{Cos}[2 \theta_{\text{s}}[t]] - 4 m \text{Cos}[\theta_{\text{ns}}[t] + \theta_{\text{s}}[t]]) \right)
\end{aligned}$$

$$\begin{aligned}
& (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - \\
& \quad \frac{1}{4} \\
& \quad m \\
& \quad \sin[\theta_{ns}[t] + \theta_s[t]]) \sin[\phi[t]] \\
& \left(\left(4096 \left(\frac{1}{128} c (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \right. \right. \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. \left. \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) \right) + \right. \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t]) \\
& \left(-c + \frac{1}{32} l (32 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 8 l m \sin[2 \theta_{ns}[t]]) \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{8} l^2 (8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) \theta_s'[t] \right) - \\
& \quad \frac{1}{131072} Kp (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. 4 l (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \right)^2 \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1 m \cos[2 \theta_{ns}[t]] + \\
& \quad \left. 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t]] \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right)^2 \phi'[t] + \frac{1}{32} \\
& \quad (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \\
& \quad (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]])) \\
& \left(-\frac{1}{8} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \right. \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. 1 (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \right. \\
& \quad \left. \sin[\theta_s[t]] \right) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2 \right) \\
& \left(-\frac{1}{8} l^2 Lp \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - \right. \\
& \quad \left. 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) + \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \right) \\
& \quad \theta_{ns}'[t] + \left(\frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta_s[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) + \\
& \quad \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad \left. 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + (5 m + 4 M) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]] + 1 (1 m \cos[2 \theta ns[t]] + \right. \\
& \left. 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t]]) \sin[\phi[t]] \right) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta ns'[t] + \frac{1}{4} l^2 \right. \\
& \left. (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[\right. \\
& \left. 2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \phi'[t] \right)^2 - \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \right. \\
& \left. (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \left. 1 (1 m \cos[2 \theta ns[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \right. \\
& \left. \sin[\theta s[t]]) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2 \right)^2 \\
& \left(2 c Kp (-d + \phi[t]) + 2 c \phi'[t] - \frac{1}{16} l (32 l m \cos[\theta s[t]] \sin[\theta ns[t]] - \right. \\
& \left. 8 l m \sin[2 \theta ns[t]]) \theta ns'[t] \phi'[t] - \frac{1}{4} l^2 (8 m \cos[\theta ns[t]] \sin[\theta s[t]] - \right. \\
& \left. 2 (5 m + 4 M) \sin[2 \theta s[t]]) \theta s'[t] \phi'[t] - \frac{1}{4} l^2 Lp \cos[\phi[t]] \right. \\
& \left. (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \omega'[t] - \right. \\
& \left. \frac{1}{2} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \theta ns'[t] \omega'[t] + \right. \\
& \left. \frac{1}{4} l^2 (2 m \cos[2 \theta ns[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \theta ns'[t] \omega'[t] + \right. \\
& \left. \frac{1}{2} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \theta s'[t] \omega'[t] + \frac{1}{4} l^2 \right. \\
& \left. (2 (5 m + 4 M) \cos[2 \theta s[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \theta s'[t] \omega'[t] + \right. \\
& \left. \frac{1}{16} (16 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \sin[\phi[t]] + 2 \cos[\phi[t]] \right. \\
& \left. (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \cos[2 \theta s[t]] + \right. \\
& \left. 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + 1 (1 m \cos[2 \theta ns[t]] + 1 (5 m + 4 M) \cos[\right. \\
& \left. 2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \sin[\phi[t]] \omega'[t]^2 \right) \Big) \Big) \Big) \Big) \Big) / \\
& \left((4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta ns[t]] + 4 l (-8 m \cos[\right. \\
& \left. \theta ns[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]])) \right) \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \\
& \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + 1 (1 m \cos[2 \theta ns[t]] + \\
& 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2)^2 - \\
& \left(4096 \left(-\frac{1}{32} (4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta ns[t]] + \right. \right. \\
& \left. \left. 4 l (-8 m \cos[\theta ns[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]])) \right) \right) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta ns'[t] + \right. \\
& \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right.
\end{aligned}$$

$$\begin{aligned}
& (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \left(-\frac{1}{4} \right. \\
& \left. 1^2 \cos[\phi[t]] (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \left. \left(\frac{1}{16} (-\cos[\phi[t]])^2 (1^2 (-2m \sin[2 \theta_{ns}[t]] + 8m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \right. \\
& \quad \left. \left. 1 (-2l m \sin[2 \theta_{ns}[t]] + 8l m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \right. \\
& \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_{ns}'[t] + \frac{1}{16} \\
& \quad \left(-\cos[\phi[t]]^2 (1^2 (8m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2(5m + 4M) \sin[2 \theta_s[t]]) + \right. \\
& \quad \left. 1 (8l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2l(5m + 4M) \sin[2 \theta_s[t]]) \right) - \\
& \quad \left. 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \right) \theta_s'[t] \Big) + \\
& \frac{1}{16} (16 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + (5m + 4M) \\
& \quad \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + 1 (1m \cos[2 \theta_{ns}[t]] + \\
& \quad 1 (5m + 4M) \cos[2 \theta_s[t]] + 8l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]]) \\
& \left(-r (-s + \omega[t]) + \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] - \right. \\
& \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \Big) + \\
& \frac{1}{256} (4 l^2 (13m + 6M) + 1 (-4l(7m + 2M) + 4l m \cos[2 \theta_{ns}[t]] + 4l \\
& \quad (-8m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5m + 4M) \cos[2 \theta_s[t]))) \\
& (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta_{ns}[t]] + \\
& \quad (5m + 4M) \cos[2 \theta_s[t]] + 8m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad 1 (1m \cos[2 \theta_{ns}[t]] + 1 (5m + 4M) \cos[2 \theta_s[t]] + 8l m \sin[\theta_{ns}[t]] \\
& \quad \sin[\theta_s[t])) + 8 l^2 (3m + 2M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]]^2) \\
& \left(\left(\left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2m \cos[2 \theta_{ns}[t]] - \right. \right. \right. \\
& \quad \left. \left. 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_{ns}'[t] + \right. \\
& \quad \left(-\frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \\
& \quad \left. (2(5m + 4M) \cos[2 \theta_s[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \theta_s'[t] \Big) \\
& \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \\
& \quad \left. \frac{1}{4} l^2 (5m + 4M - 2m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] \right) - \frac{1}{8} l^2 \cos[\phi[t]] \\
& (m \sin[2 \theta_{ns}[t]] + (5m + 4M) \sin[2 \theta_s[t]] - 4m \sin[\theta_{ns}[t] + \theta_s[t]]) \\
& \left(\frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_{ns}'[t]^2 - \frac{1}{2} l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t]^2 - \frac{1}{4} l^2 (2m \cos[2 \theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right. \\
& \quad \left. \phi[t] \theta_s'[t]^2 - \frac{1}{4} l^2 (2m \cos[2 \theta_{ns}[t]] - 4m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right)
\end{aligned}$$

$$\begin{aligned}
& \phi[\\
& \quad t]] \\
& \left(-\frac{1}{4} l^2 m (5 m + 4 M) (-1 + 2 \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \right. \\
& \quad \left. \operatorname{Sin}[\phi[t]] + \frac{1}{2} \right. \\
& \quad l^2 \\
& \quad m \\
& \quad \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]] \\
& \quad \left. (5 m + 4 M - 2 m \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \right. \\
& \quad \left. \operatorname{Sin}[\phi[t]] \right) \\
& \left(-p \theta_{ns}'[t] \left(-E2Dref + \frac{1}{8} l (4 g (-m \operatorname{Cos}[\text{Beta} - \theta_{ns}[t]] + (3 m + 2 M) \operatorname{Cos}[\text{Beta} - \theta_s[t]]) + \right. \right. \\
& \quad \left. \left. l (m \theta_{ns}'[t]^2 - 4 m \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]] \theta_{ns}'[t] \theta_s'[t] + (5 m + 4 M) \theta_s'[t]^2) \right) \right) - \\
& \left(65536 \left(\frac{1}{4096} c^2 l (32 l m \operatorname{Cos}[\theta_s[t]] \operatorname{Sin}[\theta_{ns}[t]] - 8 l m \operatorname{Sin}[2 \theta_{ns}[t]]) \right. \right. \\
& \quad \left(-\operatorname{Cos}[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \operatorname{Cos}[2 \theta_{ns}[t]] + (5 m + 4 M) \right. \\
& \quad \left. \operatorname{Cos}[2 \theta_s[t]] + 8 m \operatorname{Sin}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]]) + l (1 m \operatorname{Cos}[2 \theta_{ns}[t]] + \right. \\
& \quad \left. l (5 m + 4 M) \operatorname{Cos}[2 \theta_s[t]] + 8 l m \operatorname{Sin}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]]) \right) + \\
& \quad \left. 8 l^2 (3 m + 2 M - 2 m \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \operatorname{Sin}[\phi[t]]^2 \right)^2 (-d + \phi[t])^2 + \\
& \quad \frac{1}{1024} (4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \operatorname{Cos}[2 \theta_{ns}[t]] + 4 l \\
& \quad (-8 m \operatorname{Cos}[\theta_{ns}[t]] \operatorname{Cos}[\theta_s[t]] + (5 m + 4 M) \operatorname{Cos}[2 \theta_s[t]])) \right)^2 \\
& \quad \left(-2 \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \operatorname{Sin}[\phi[t]] \theta_{ns}'[t] + \right. \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \operatorname{Sin}[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \operatorname{Cos}[\phi[t]] \right. \\
& \quad \left. (m \operatorname{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \operatorname{Sin}[2 \theta_s[t]] - 4 m \operatorname{Sin}[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \\
& \quad \left(\frac{1}{16} (-\operatorname{Cos}[\phi[t]]^2 (l^2 (-2 m \operatorname{Sin}[2 \theta_{ns}[t]] + 8 m \operatorname{Cos}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]]) + \right. \\
& \quad \left. l (-2 l m \operatorname{Sin}[2 \theta_{ns}[t]] + 8 l m \operatorname{Cos}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]])) \right) + \\
& \quad 16 l^2 m \operatorname{Sin}[\theta_{ns}[t] - \theta_s[t]] \operatorname{Sin}[\phi[t]]^2 \left(-r (-s + \omega[t]) - \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \operatorname{Sin}[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \operatorname{Cos}[\phi[t]] \right. \\
& \quad \left. (m \operatorname{Sin}[2 \theta_{ns}[t]] + (5 m + 4 M) \operatorname{Sin}[2 \theta_s[t]] - 4 m \operatorname{Sin}[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \phi'[t] \right) - \frac{1}{4} l^2 m (-1 + 2 \operatorname{Cos}[\theta_{ns}[t] - \theta_s[t]]) \operatorname{Sin}[\phi[t]] \\
& \quad \left(\frac{1}{16} (-\operatorname{Cos}[\phi[t]]^2 (l^2 (-2 m \operatorname{Sin}[2 \theta_{ns}[t]] + 8 m \operatorname{Cos}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]]) + \right. \\
& \quad \left. l (-2 l m \operatorname{Sin}[2 \theta_{ns}[t]] + 8 l m \operatorname{Cos}[\theta_{ns}[t]] \operatorname{Sin}[\theta_s[t]])) \right) + \\
& \quad 16 l^2 m \operatorname{Sin}[\theta_{ns}[t] - \theta_s[t]] \operatorname{Sin}[\phi[t]]^2 \theta_{ns}'[t] + \frac{1}{8} (-\operatorname{Cos}[\phi[t]]^2 \\
& \quad (l^2 (8 m \operatorname{Cos}[\theta_s[t]] \operatorname{Sin}[\theta_{ns}[t]] - 2 (5 m + 4 M) \operatorname{Sin}[2 \theta_s[t])) +
\end{aligned}$$

$$\begin{aligned}
& 1 (8 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 2 l (5 m + 4 M) \sin[2 \theta s[t]]) - \\
& 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2) \theta s'[t] + \\
& \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& 1 (1 m \cos[2 \theta ns[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + \\
& 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \sin[\phi[t]] \phi'[t]) + \\
& \frac{1}{4} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& 1 (1 m \cos[2 \theta ns[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \\
& \sin[\theta s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t])) \\
& \sin[\phi[t]]^2) \left(\left(-1^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta s'[t] + \right. \right. \\
& \left. \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] + \right. \right. \\
& \left. \left. \frac{1}{8} l^2 (2 m \cos[2 \theta ns[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) \\
& \left(r (-s + \omega[t]) + \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right. \\
& \left. \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2 \theta ns[t]] + (5 m + 4 M) \sin[2 \theta s[t]] - \right. \\
& \left. 4 m \sin[\theta ns[t] + \theta s[t])) \phi'[t] \right) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \\
& \sin[\phi[t]] \left(\frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t]^2 - \right. \\
& \left. \frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta s'[t]^2 - \frac{1}{2} l^2 m (-1 + \right. \\
& \left. 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \theta ns'[t] \phi'[t] + \frac{1}{8} l^2 (m \sin[2 \theta ns[t]] + \right. \\
& \left. (5 m + 4 M) \sin[2 \theta s[t]] - 4 m \sin[\theta ns[t] + \theta s[t]]) \sin[\phi[t]] \phi'[t]^2 + \right. \\
& \left. \theta s'[t] \left(-1^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] + \left(\frac{1}{4} l^2 (5 m + 4 M - \right. \right. \right. \\
& \left. \left. 2 m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2 (5 m + 4 M) \cos[2 \theta s[\right. \right. \\
& \left. \left. t] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) + r \omega'[t] \left. \right) + \\
& \frac{1}{256} (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& 1 (1 m \cos[2 \theta ns[t]] + 1 (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \\
& \sin[\theta s[t])) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t])) \sin[\phi[t]]^2)^2 \\
& \left(-2 l^2 m \sin[\theta ns[t] - \theta s[t]] \theta s'[t]^2 - 4 l^2 m \sin[\theta ns[t] - \theta s[t]] \right. \\
& \left. \sin[\phi[t]] \theta s'[t] \omega'[t] - 2 \left(g l m \sin[\text{Beta} - \theta ns[t]] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{32} l (32 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 8 l m \sin[2 \theta ns[t]]) \phi'[t]^2 + \\
& 2 \left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \\
& \quad \left. (2 m \cos[2 \theta ns[t]] - 4 m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] + \\
& \frac{1}{16} \left(-\cos[\phi[t]]^2 (l^2 (-2 m \sin[2 \theta ns[t]] + 8 m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad \left. l (-2 l m \sin[2 \theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t])) + \right. \\
& \quad \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \right) \omega'[t]^2 \Big) \Big) \Big) \Big) \Big) / \\
& \left((4 l^2 (13 m + 6 M) + l (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta ns[t]] + 4 l (-8 m \cos[\right. \\
& \quad \left. \theta ns[t]] \cos[\theta s[t]] + (5 m + 4 M) \cos[2 \theta s[t]])) \right)^2 \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + l (l m \cos[2 \theta ns[t]] + \\
& \quad l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2) \Big) \Big) \Big) \Big) / \\
& (m (-3 m - 4 M + 2 m \cos[2 (\theta ns[t] - \theta s[t])]) (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + \\
& \quad l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t]]) + \\
& \quad l (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2) + \\
& \left(32 \left(-l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right. \right. \\
& \quad \left(-\frac{1}{4} l^2 m (5 m + 4 M) (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] + \right. \\
& \quad \left. \left. \frac{1}{2} l^2 m \cos[\theta ns[t] - \theta s[t]] (5 m + 4 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]] \right) \right) + \\
& \quad \frac{1}{16} l^2 m (-3 m - 4 M + 2 m \cos[2 (\theta ns[t] - \theta s[t])]) \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + \\
& \quad (5 m + 4 M) \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad l (l m \cos[2 \theta ns[t]] + l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2) \Big) \\
& \left(-p \theta ns'[t] \left(-E2Dref + \frac{1}{8} l (4 g (-m \cos[\text{Beta} - \theta ns[t]] + (3 m + 2 M) \cos[\text{Beta} - \theta s[t]]) + \right. \right. \\
& \quad \left. \left. l (m \theta ns'[t]^2 - 4 m \cos[\theta ns[t] - \theta s[t]] \theta ns'[t] \theta s'[t] + (5 m + 4 M) \theta s'[t]^2) \right) \right) - \\
& \left(65536 \left(\frac{1}{4096} c^2 l (32 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 8 l m \sin[2 \theta ns[t]]) \right. \right. \\
& \quad (-\cos[\phi[t]]^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta ns[t]] + (5 m + 4 M) \\
& \quad \cos[2 \theta s[t]] + 8 m \sin[\theta ns[t]] \sin[\theta s[t])) + l (l m \cos[2 \theta ns[t]] + \\
& \quad l (5 m + 4 M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad \left. \left. 8 l^2 (3 m + 2 M - 2 m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2 \right)^2 (-d + \phi[t])^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{1024} \left(4 l^2 (13 m + 6 M) + 1 (-4 l (7 m + 2 M) + 4 l m \cos[2 \theta_{ns}[t]] + 4 l \right. \\
& \quad \left. (-8 m \cos[\theta_{ns}[t]] \cos[\theta_s[t]] + (5 m + 4 M) \cos[2 \theta_s[t]]) \right)^2 \\
& \left(-2 \left(r (-s + \omega[t]) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_{ns}'[t] + \right. \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \phi'[t] \right) \\
& \left(\frac{1}{16} (-\cos[\phi[t]])^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \left. l (-2 l m \sin[2 \theta_{ns}[t]] + 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \left(-r (-s + \omega[t]) - \right. \\
& \quad \left. \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \theta_s'[t] + \frac{1}{8} l^2 \cos[\phi[t]] \right. \\
& \quad \left. (m \sin[2 \theta_{ns}[t]] + (5 m + 4 M) \sin[2 \theta_s[t]] - 4 m \sin[\theta_{ns}[t] + \theta_s[t]]) \right. \\
& \quad \left. \phi'[t] \right) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \\
& \left(\frac{1}{16} (-\cos[\phi[t]])^2 (l^2 (-2 m \sin[2 \theta_{ns}[t]] + 8 m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) + \right. \\
& \quad \left. l (-2 l m \sin[2 \theta_{ns}[t]] + 8 l m \cos[\theta_{ns}[t]] \sin[\theta_s[t]]) \right) + \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_{ns}'[t] + \frac{1}{8} (-\cos[\phi[t]])^2 \\
& \quad (l^2 (8 m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 (5 m + 4 M) \sin[2 \theta_s[t]]) + \\
& \quad \left. l (8 l m \cos[\theta_s[t]] \sin[\theta_{ns}[t]] - 2 l (5 m + 4 M) \sin[2 \theta_s[t])) \right) - \\
& \quad 16 l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]]^2 \theta_s'[t] + \\
& \quad \frac{1}{8} (16 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] \sin[\phi[t]] + \\
& \quad 2 \cos[\phi[t]] (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. l (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + \right. \\
& \quad \left. 8 l m \sin[\theta_{ns}[t]] \sin[\theta_s[t])) \sin[\phi[t]] \phi'[t] \right) + \\
& \frac{1}{4} (-\cos[\phi[t]])^2 (-2 l^2 (13 m + 6 M) + l^2 (14 m + 4 M + m \cos[2 \theta_{ns}[t]] + \\
& \quad (5 m + 4 M) \cos[2 \theta_s[t]] + 8 m \sin[\theta_{ns}[t]] \sin[\theta_s[t]]) + \\
& \quad \left. l (1 m \cos[2 \theta_{ns}[t]] + 1 (5 m + 4 M) \cos[2 \theta_s[t]] + 8 l m \sin[\theta_{ns}[t]] \right. \\
& \quad \left. \sin[\theta_s[t])) \right) + 8 l^2 (3 m + 2 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \\
& \sin[\phi[t]]^2 \left(\left(-l^2 m \sin[\theta_{ns}[t] - \theta_s[t]] \sin[\phi[t]] \theta_s'[t] + \right. \right. \\
& \quad \left(-\frac{1}{4} l^2 m (-1 + 2 \cos[\theta_{ns}[t] - \theta_s[t]]) \cos[\phi[t]] + \right. \\
& \quad \left. \left. \frac{1}{8} l^2 (2 m \cos[2 \theta_{ns}[t]] - 4 m \cos[\theta_{ns}[t] + \theta_s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) \\
& \left(r (-s + \omega[t]) + \frac{1}{4} l^2 (5 m + 4 M - 2 m \cos[\theta_{ns}[t] - \theta_s[t]]) \sin[\phi[t]] \right)
\end{aligned}$$

$$\begin{aligned}
& \theta s'[t] - \frac{1}{8} l^2 \cos[\phi[t]] (m \sin[2 \theta ns[t]] + (5m + 4M) \sin[2 \theta s[t]] - \\
& \quad 4m \sin[\theta ns[t] + \theta s[t]]) \phi'[t] \Big) - \frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \\
& \sin[\phi[t]] \left(\frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t]^2 - \right. \\
& \quad \frac{1}{2} l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta s'[t]^2 - \frac{1}{2} l^2 m (-1 + \\
& \quad 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] \theta ns'[t] \phi'[t] + \frac{1}{8} l^2 (m \sin[2 \theta ns[t]] + \\
& \quad (5m + 4M) \sin[2 \theta s[t]] - 4m \sin[\theta ns[t] + \theta s[t]]) \sin[\phi[t]] \phi'[t]^2 + \\
& \quad \theta s'[t] \left(-l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]] \theta ns'[t] + \left(\frac{1}{4} l^2 (5m + 4M - \right. \right. \\
& \quad \left. \left. 2m \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 (2(5m + 4M) \cos[2 \theta s[\right. \right. \\
& \quad \left. \left. t] - 4m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \right) + r \omega'[t] \Big) \Big) + \\
& \frac{1}{256} \left(-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta ns[t]] + \right. \\
& \quad (5m + 4M) \cos[2 \theta s[t]] + 8m \sin[\theta ns[t]] \sin[\theta s[t]]) + \\
& \quad 1 (1m \cos[2 \theta ns[t]] + 1 (5m + 4M) \cos[2 \theta s[t]] + 8lm \sin[\theta ns[t]] \\
& \quad \sin[\theta s[t])) + 8 l^2 (3m + 2M - 2m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2 \Big)^2 \\
& \left(-2 l^2 m \sin[\theta ns[t] - \theta s[t]] \theta s'[t]^2 - 4 l^2 m \sin[\theta ns[t] - \theta s[t]] \right. \\
& \quad \sin[\phi[t]] \theta s'[t] \omega'[t] - 2 \left(g l m \sin[\text{Beta} - \theta ns[t]] + \right. \\
& \quad \left. \frac{1}{32} l (32 l m \cos[\theta s[t]] \sin[\theta ns[t]] - 8 l m \sin[2 \theta ns[t]]) \phi'[t]^2 + \right. \\
& \quad \left. 2 \left(\frac{1}{4} l^2 m (-1 + 2 \cos[\theta ns[t] - \theta s[t]]) \cos[\phi[t]] - \frac{1}{8} l^2 \right. \right. \\
& \quad \left. \left. (2m \cos[2 \theta ns[t]] - 4m \cos[\theta ns[t] + \theta s[t]]) \cos[\phi[t]] \right) \phi'[t] \omega'[t] + \right. \\
& \quad \left. \frac{1}{16} (-\cos[\phi[t]]^2 (l^2 (-2m \sin[2 \theta ns[t]] + 8m \cos[\theta ns[t]] \sin[\theta s[t]]) + \right. \\
& \quad \left. 1 (-2 l m \sin[2 \theta ns[t]] + 8 l m \cos[\theta ns[t]] \sin[\theta s[t])) + \right. \\
& \quad \left. \left. 16 l^2 m \sin[\theta ns[t] - \theta s[t]] \sin[\phi[t]]^2 \right) \omega'[t]^2 \right) \Big) \Big) \Big) \Big) / \\
& \left(\left(4 l^2 (13m + 6M) + 1 (-4 l (7m + 2M) + 4 l m \cos[2 \theta ns[t]] + 4 l (-8m \cos[\right. \right. \\
& \quad \left. \left. \theta ns[t]] \cos[\theta s[t]] + (5m + 4M) \cos[2 \theta s[t]] \right) \right) \Big)^2 \\
& \left(-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + l^2 (14m + 4M + m \cos[2 \theta ns[t]] + (5m + 4M) \right. \\
& \quad \cos[2 \theta s[t]] + 8m \sin[\theta ns[t]] \sin[\theta s[t]]) + 1 (1m \cos[2 \theta ns[t]] + \\
& \quad 1 (5m + 4M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad \left. 8 l^2 (3m + 2M - 2m \cos[\theta ns[t] - \theta s[t]]) \sin[\phi[t]]^2 \right) \Big) \Big) \Big) / \\
& \left(l^2 m (-3m - 4M + 2m \cos[2 (\theta ns[t] - \theta s[t])]) (-\cos[\phi[t]]^2 (-2 l^2 (13m + 6M) + \right. \\
& \quad l^2 (14m + 4M + m \cos[2 \theta ns[t]] + (5m + 4M) \cos[2 \theta s[t]] + 8m \sin[\theta ns[t]] \sin[\theta s[t])) + \\
& \quad \left. 1 (1m \cos[2 \theta ns[t]] + 1 (5m + 4M) \cos[2 \theta s[t]] + 8 l m \sin[\theta ns[t]] \sin[\theta s[t])) \right) +
\end{aligned}$$