

# Sensing and Control with Limited Transmissions

(based on joint work with O.C. Imer)

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CASY Workshop -- Bertinoro

**May 22-26, 2006**

# Outline

- Networks & Control
- Joint sensor-controller design
  - What/when/how to transmit & control
- Estimation with “Power-Limited Communication”
- Control with “Power-Limited Communication”
- When to measure & when to control
- Other issues / Conclusions

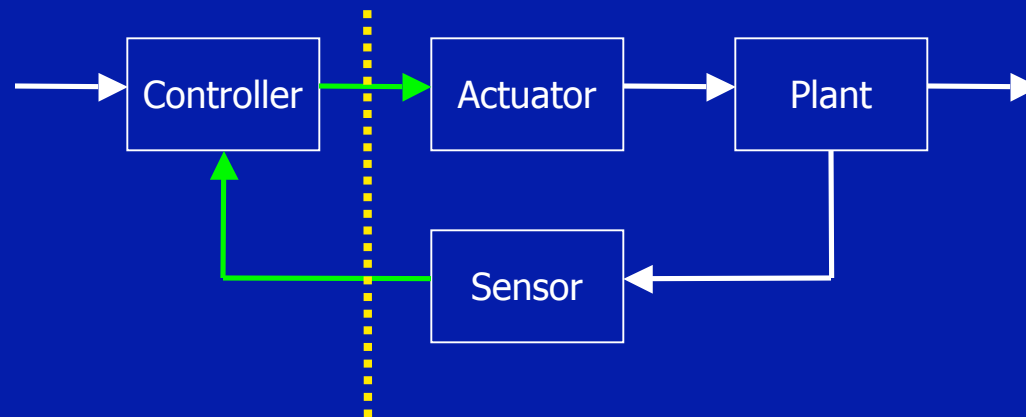
# Networks & Control

- A surge of research interest in using networks for communication in control/monitoring systems
- **Advantages:** flexibility, reduced wiring, lower installation costs, agility in diagnosis and maintenance, remote operations, ...
- **Challenges:** synchronization, timing problems, reliability, *communication network and channel constraints*, ...

# Networks & Control

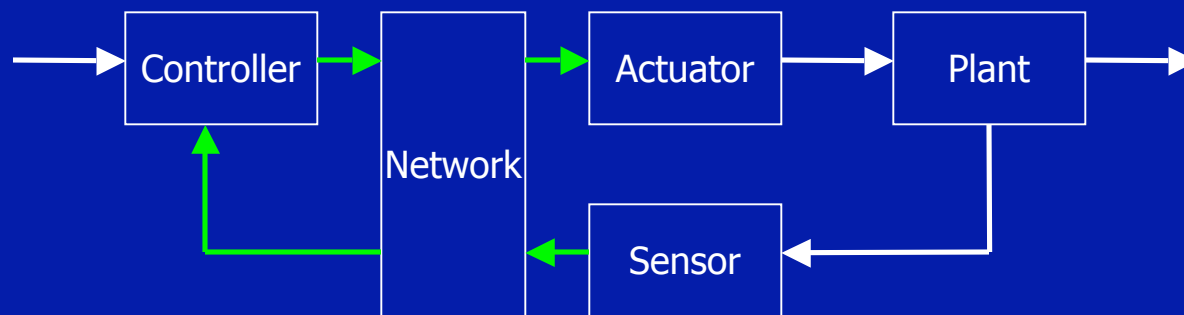
- Traditional Control Systems have **dedicated data paths** for communication

## Controller-Plant Communication

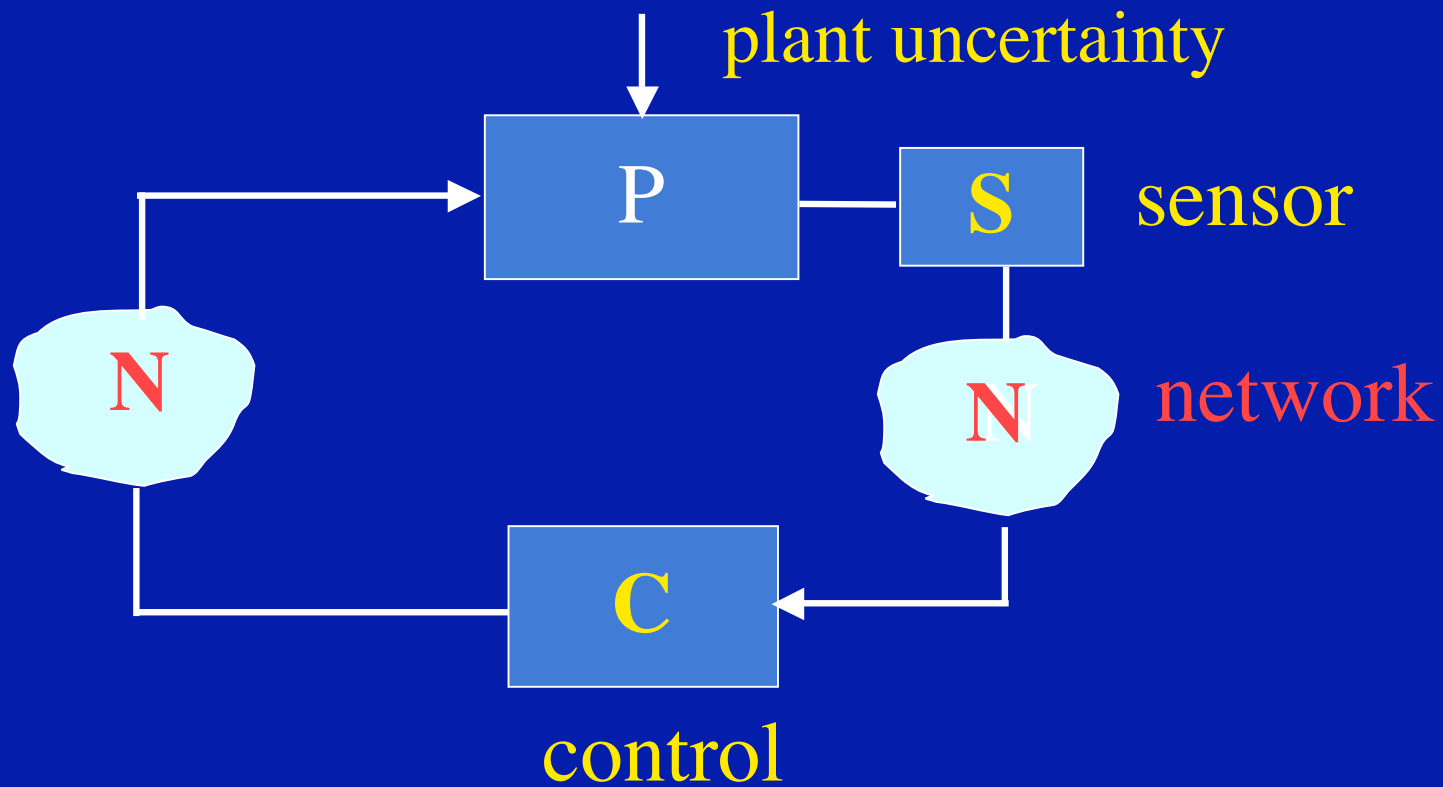


# Networks & Control

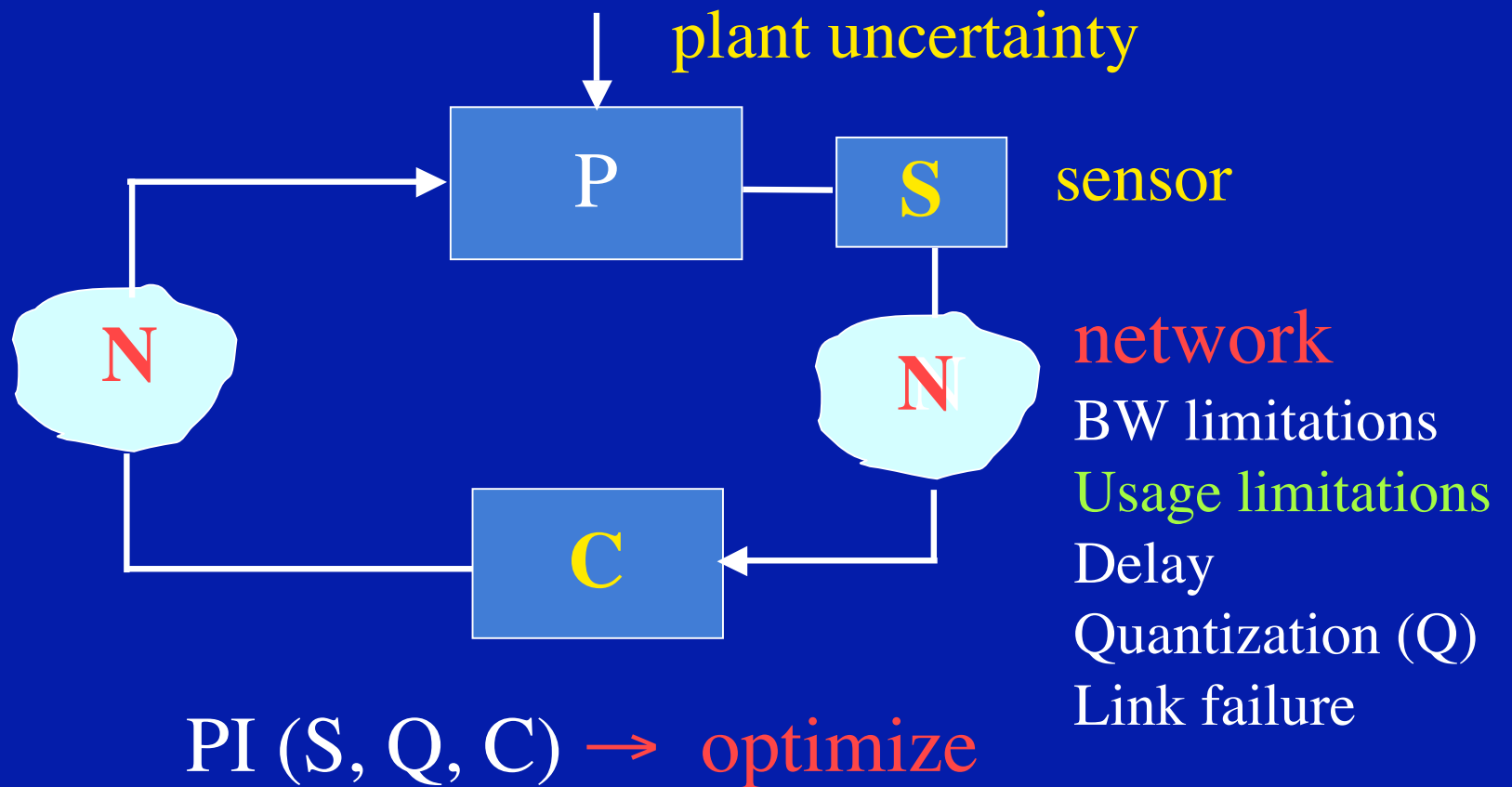
- In Networked Control Systems (NCS) **controller-plant communication** takes place over a **network**



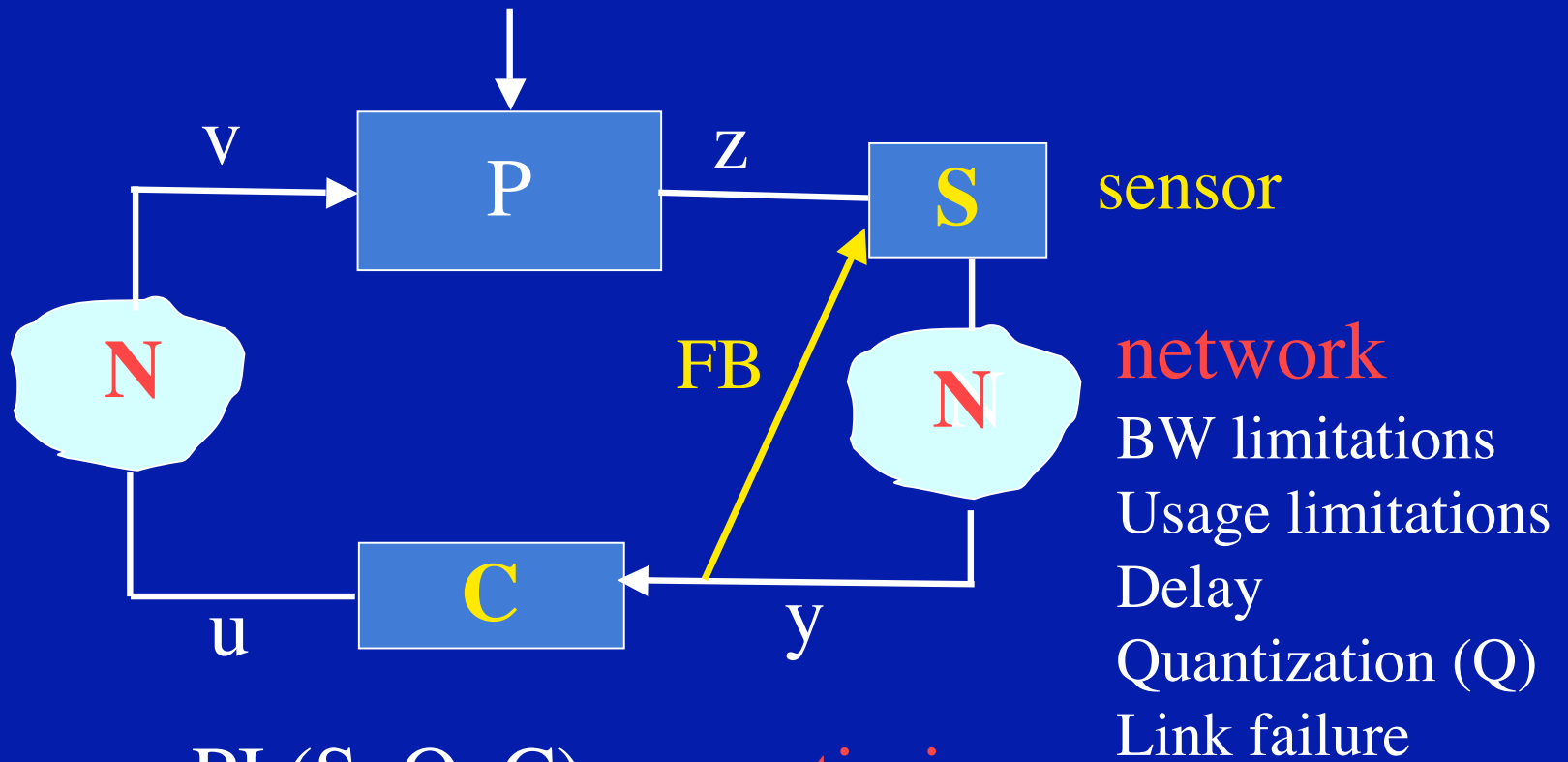
# Remote Control Paradigm



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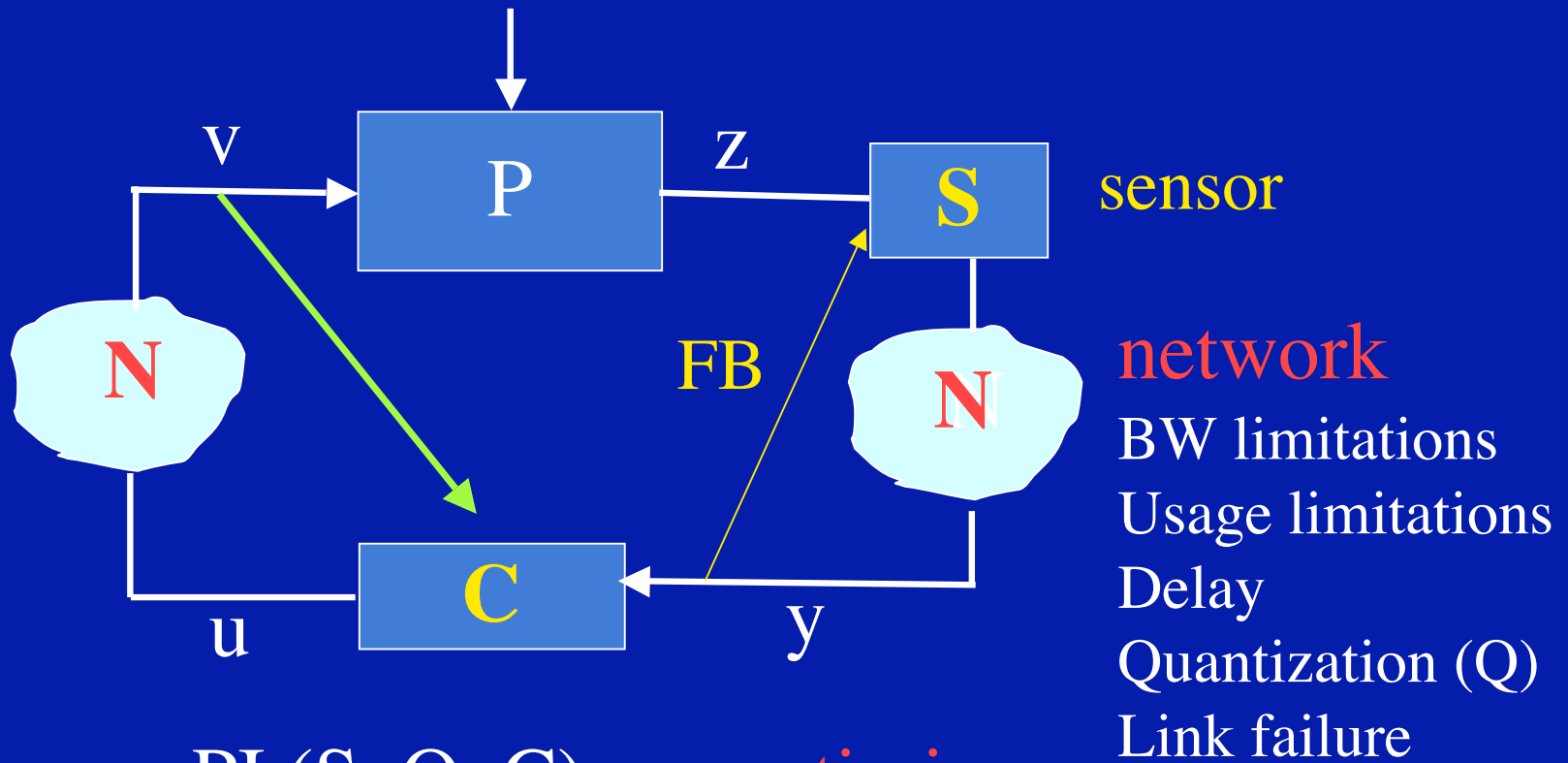


# Remote Control Paradigm



PI (S, Q, C)  $\rightarrow$  optimize  
**Non-classical information!**

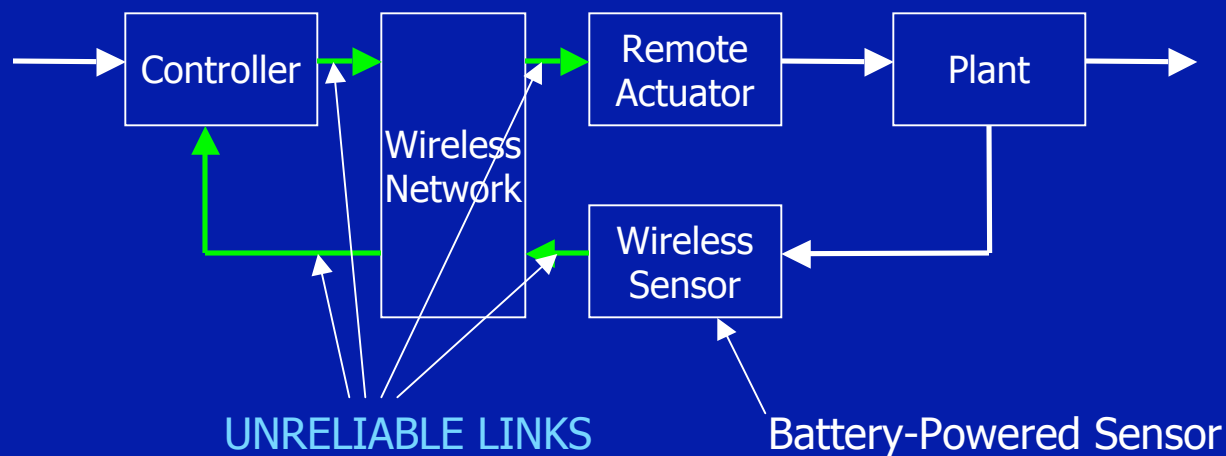
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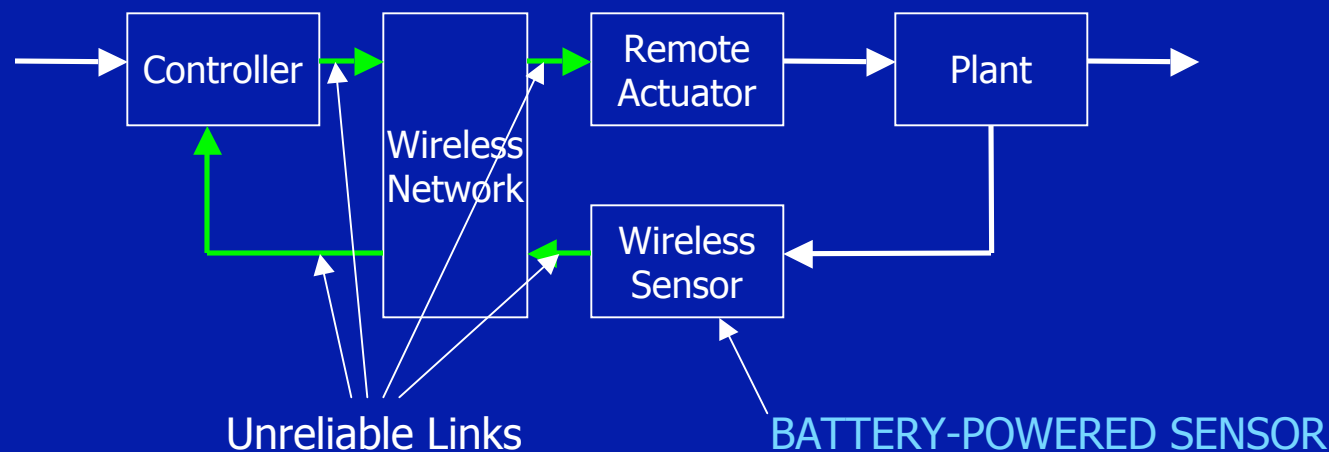
# Wireless Networks & Control

- Estimation & Control when **controller-plant communication** takes place over a **Wireless Network**



# Wireless Networks & Control

- Estimation & Control when **controller-plant communication** takes place over a **Wireless Network**



# Limited Usage in Sensing & Control

## Reasons

- To conserve battery power in wireless sensors  
(with RF channels a significant amount of power is consumed to transmit sensor measurements)
- Sharing of bandwidth among several users  
(time allocation limited transmission -- TDM systems)
- Power limitation on controller
- Limits on the frequency of interaction with the plant/system
- CANs -- actuator, controller, sensor connected over a serial bus -- access one at a time

# Limited Usage in Sensing & Control

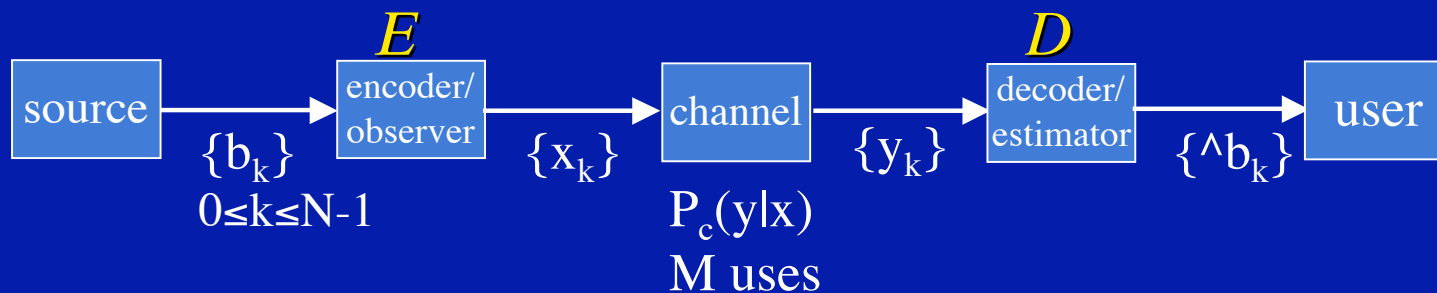
## Reasons

- To conserve battery power in wireless sensors (with RF channels a significant amount of power is consumed to transmit sensor measurements)
- Sharing of channels with other sensors (time allocation)
- Power consumption
- Limits on the frequency of interaction with the plant/system
- CANs -- actuator, controller, sensor connected over a serial bus -- access one at a time

**Sensing and Control  
are Expensive !**

# PROBLEM CLASS I

## Optimal Estimation over a Limited-Use Channel

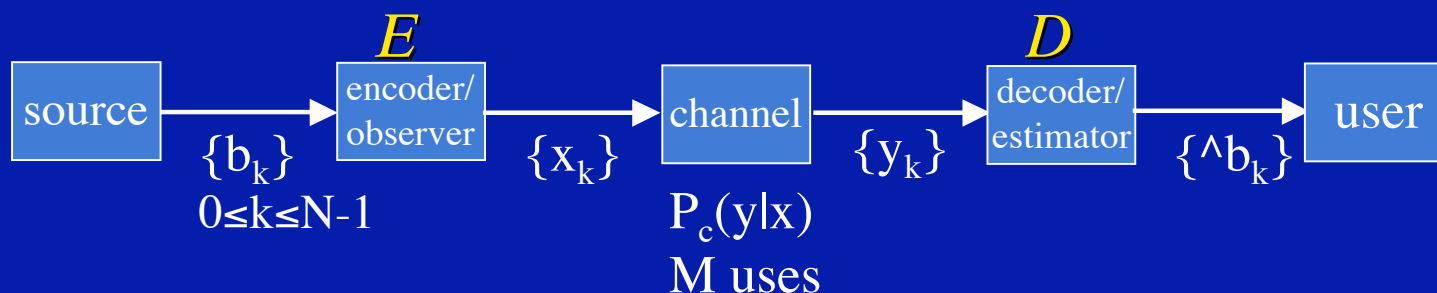


$$x_k = E(z_k) \quad x \in X \quad y \in Y$$

$$z_k = b_k + v_k \quad M < N$$

*Given a “source” and a “memoryless channel”, for a given message length  $N$ , and number of channel uses  $M$ , what is the minimum attainable value of the average distortion  $D_{(M,N)}$  and a corresponding  $E$  &  $D$  pair?*

## Optimal Estimation over a Limited-Use Channel



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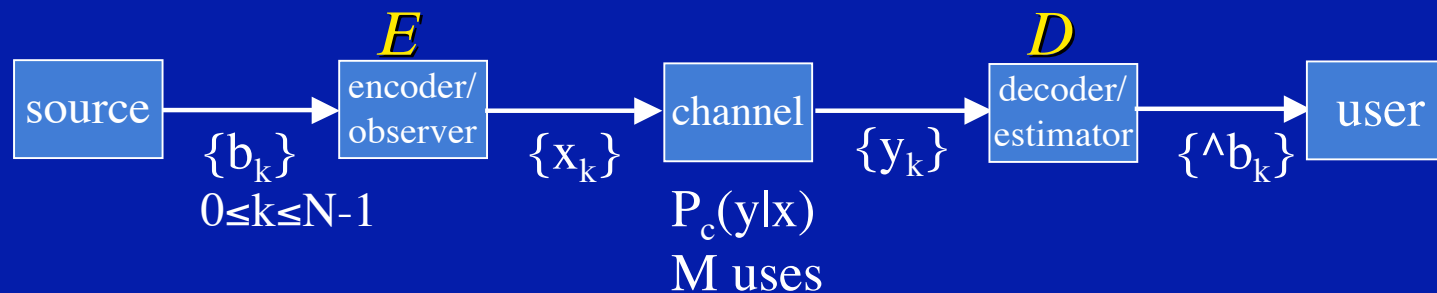
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e.g.  $D_{(M,N)} = E\left\{\frac{1}{N} \sum_{k=0}^{N-1} (b_k - \hat{b}_k)^2\right\}$  MSE

or the probability of error measure

## Optimal Estimation over a Limited-Use Channel



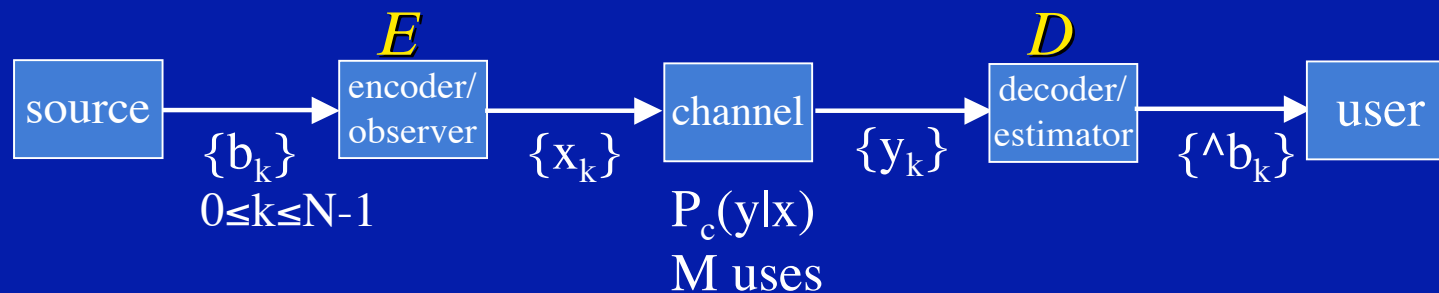
$$x_k = E(z_k) \quad x \in X \quad y \in Y$$

$$z_k = b_k + v_k \quad M < N$$

### Order of actions at time $k$ :

1.  $b_k$  (or  $z_k$ ) becomes available to the sensor
2. Sensor makes a decision: transmit/shape or not
3. Estimator acts by generating  $\hat{b}_k$
4. Estimation error is incurred and we move to  $k+1$

## Optimal Estimation over a Limited-Use Channel



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$$x \in X \quad y \in Y$$

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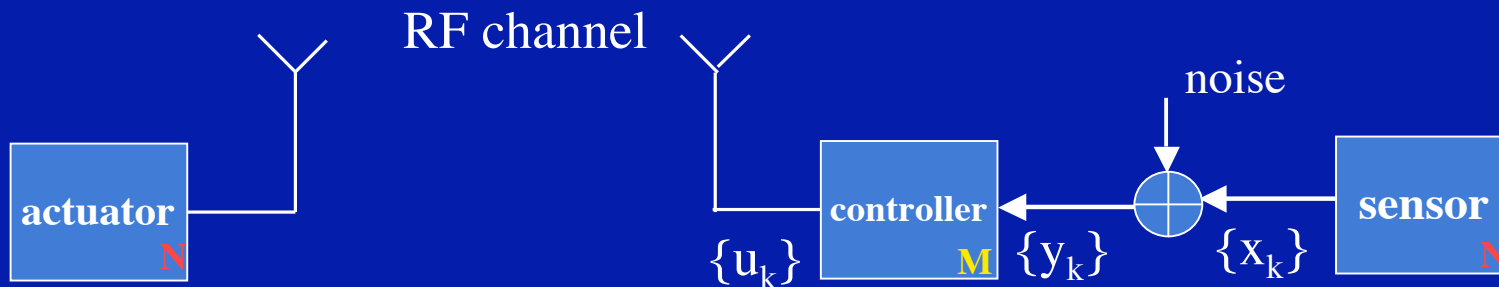
**Dynamic &  
Non-classical**

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# PROBLEM CLASS II

## Control over a Limited-Use Channel



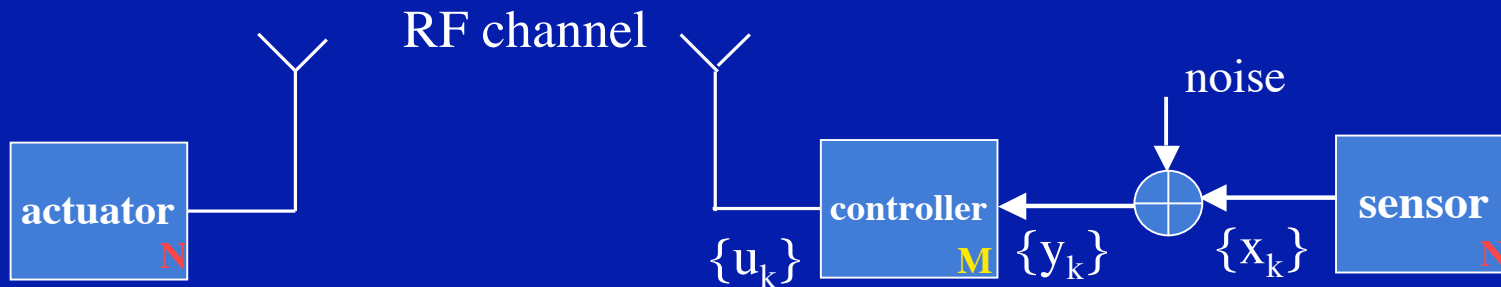
$$x_{k+1} = f(x_k, u_k, w_k) \quad x_k \in X_k, u_k \in U_k, y \in Y_k$$

$$y_k = h_k(x_k) + v_k \quad M < N$$

$$u_k = \mu_k(I_k), I_k := \{y_{[0,k]}, u_{[0,k-1]}\}, \mu_k \rightarrow U_k \text{ only } M \text{ times}$$

*Given a horizon of  $N$  units, and with controller allowed to transmit for only  $M < N$  times, what is the minimum attainable value of a performance index  $J$ , and a corresponding controller?*

# Control over a Limited-Use Channel



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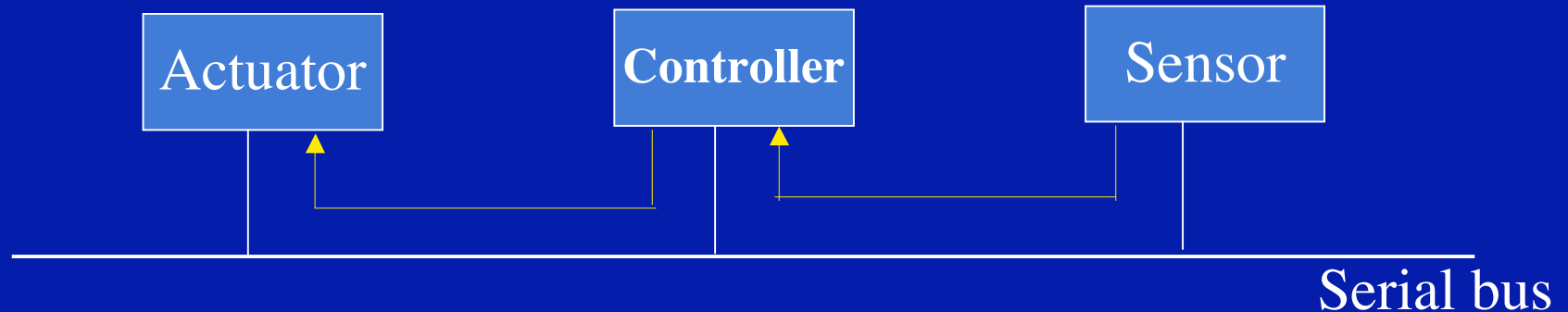
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*Given a horizon of  $N$  units, and with control allowed to transmit for only  $M < N$  times, what is the minimum attainable value of a performance index  $J$ , and a corresponding controller?*

e.g.  $J = E\{q(x_N) + \sum_0^{N-1} g(x_k, u_k)\}$

# PROBLEM CLASS III

## Scheduled Measurements & Controls



$$x_{k+1} = A x_k + a_k u_k + w_k, \quad k = 0, \dots, N-1$$

$a_k$  is 0 or 1

$$y_k = (1 - a_k) x_k, \quad k = 0, \dots, N-1 \quad \text{measurement}$$

Controller cannot receive and send simultaneously

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# PROBLEM CLASS I

## OPTIMAL ESTIMATION

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# Estimation: A Special Case

$N=2, M=1, b_0, b_1$  i.i.d. Gaussian, 0-mean, variance  $\sigma^2$

Perfect channel, no noise

Estimation error:  $e = E \{ (b_0 - \hat{b}_0)^2 + (b_1 - \hat{b}_1)^2 \}$

**Open-loop sensor policy:**

Arbitrarily picks transmission time  $\implies e_{OL} = \sigma^2$

**Closed-loop sensor policy:**

Transmit  $b_0$  if it lies outside  $[\alpha, \beta]$ ,  $\alpha < 0 < \beta$ ; otherwise  $b_1$

Minimization problem faced by sensor:

$$e_{(\alpha, \beta)} = \int_{\alpha}^{\beta} (b - E[b \mid b \in [\alpha, \beta]])^2 f(b) db + \sigma^2 P\{b_0 \notin [\alpha, \beta]\}$$

# Special Case: Solution

$$(\alpha^*, \beta^*) = (-\sigma, \sigma)$$



$$e_{\text{CL}}^* = e_{(\alpha^*, \beta^*)} = [1 - \sqrt{(2 / \pi e)}] \sigma^2 \\ \approx 0.52 \sigma^2$$



**48% improvement over the OL policy**

# Special Case: Solution

$$(\alpha^*, \beta^*) = (-\sigma, \sigma)$$



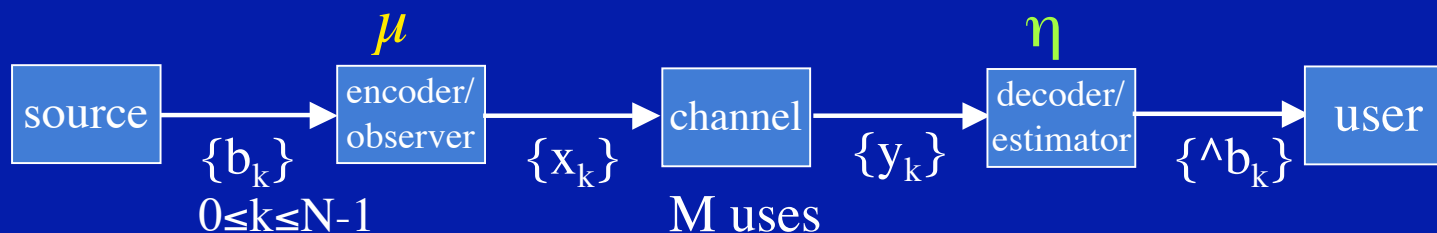
**The knowledge of no action  
is useful information !!**



**48% improvement over the OL policy**

# Optimal Estimation over a Limited-Use Channel

-- i.i.d. case, perfect channel --



$$x_k = \mu_k(I_k^e) \quad \hat{b}_k = \eta_k(I_k^d) \quad z_k = b_k + v_k$$

$s_k$  : # channel uses left at time  $k$

$t_k$  : # decision slots left at time  $k$

$$I_k^e = \{(s_k, t_k); z_{[0,k]}, x_{[0,k-1]}\}, \quad 1 \leq k \leq N-1; \quad I_0^e = \{(s_0, t_0); z_0\}$$

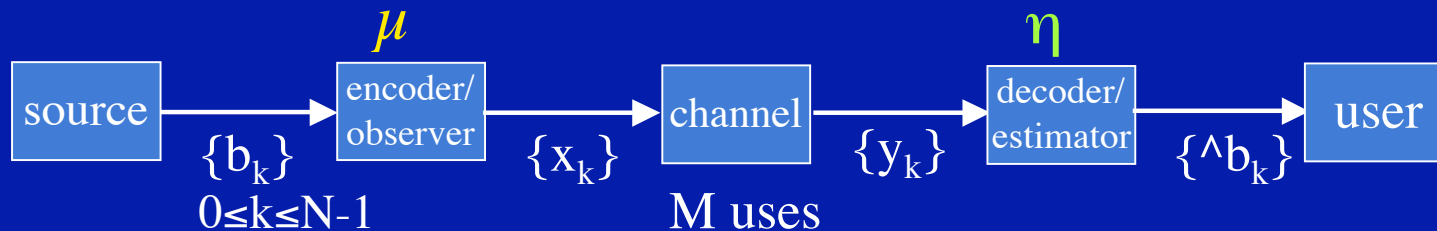
$$I_k^d = \{(s_k, t_k); y_{[0,k]}\}, \quad 0 \leq k \leq N-1$$

$\rho_k$  : 0-1 quantity; 1 if sensor transmits

$$s_{k+1} = s_k - \rho_k, \quad s_0 = M \quad \textit{s-dynamics}$$

# Optimal Estimation over a Limited-Use Channel

-- i.i.d. case, perfect channel --



$$x_k = \mu_k(I_k^e) \quad \hat{b}_k = \eta_k(I_k^d) \quad z_k = b_k + v_k$$

Find  $\mu_{[0, N-1]}$  and  $\eta_{[0, N-1]}$  that minimize a given PI: MSE or Probability of error

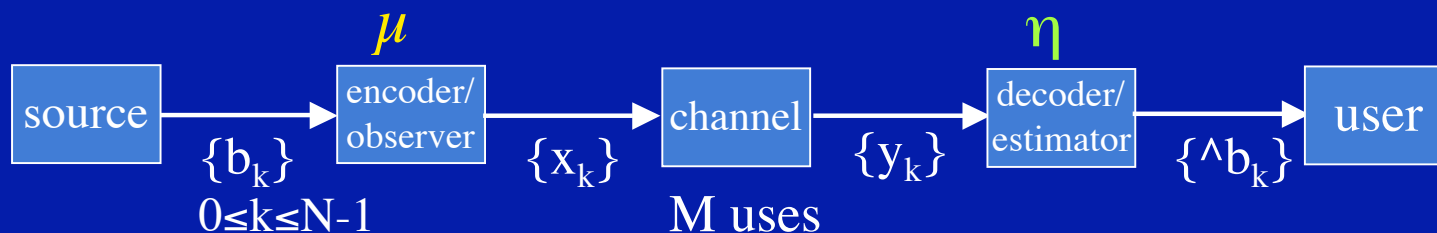
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## Optimal Estimation over a Limited-Use Channel

-- i.i.d. case, perfect channel --



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$$z_k = b_k + v_k$$

Find  $\mu_{[0, N-1]}$  and  $\eta_{[0, N-1]}$  that minimize a given PI: MSE or Probability of error

- Best choice for  $\eta_k$  is  $E[b_k | (s_k, t_k); x_k]$  or MAP
- Sufficient statistics for optimum  $\mu_k$  is

$$S_k^e := \{(s_k, t_k); z_k\}$$

# Solution

Best sensor policy is of the form:

At time  $k$  transmit  $z_k$  if it is in a measurable set  $\mathbb{Y}(s_k, t_k)$ ,  
otherwise do not

$\mathbb{Y}(s, t)$  obtained offline as the minimizer in a recursive equation  
satisfied by accumulated optimum error,  $e^*(s, t)$ , at each point  $(s, t)$ :

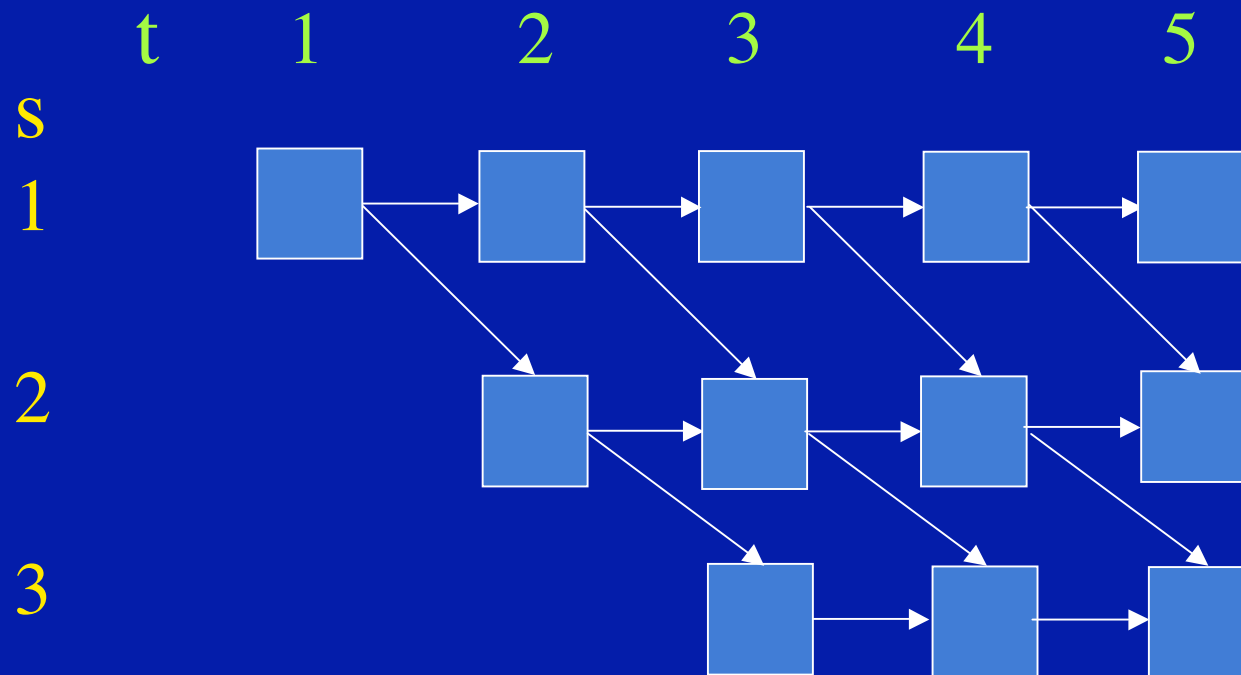
$$e^*(s, t) = \min_{\mathbb{Y}(s, t)} \left\{ e^*(s-1, t-1) \text{Prob}(z \in \mathbb{Y}) + e^*(s, t-1) \text{Prob}(z \notin \mathbb{Y}) \right. \\ \left. + \text{average error at } (s, t) \text{ due to decision at } (s, t) \right\}$$

$$e^*(t, t) = 0, e^*(0, t) = t \text{ var}(b)$$

Specific structure of  $\mathbb{Y}(s, t)$  depends on the pdf/pmf and PI.

# Offline Computations

The recursion can be solved offline, and the optimal sets  $\Upsilon(s,t)$  tabulated starting with small values of  $(s,t)$



# Explicit Solution in a Special Case

Continuous distribution  $f$  for  $b$ ,  $f(-b) = f(b)$

No noise  $v$  (from source to sensor) --n.l.o.g

$$\Rightarrow \mathbb{Y}^c(s,t) = [-\beta_{(s,t)}, \beta_{(s,t)}]$$

$$\beta_{(s,t)} = \sqrt{\{e^*(s-1, t-1) - e^*(s, t-1)\}}$$

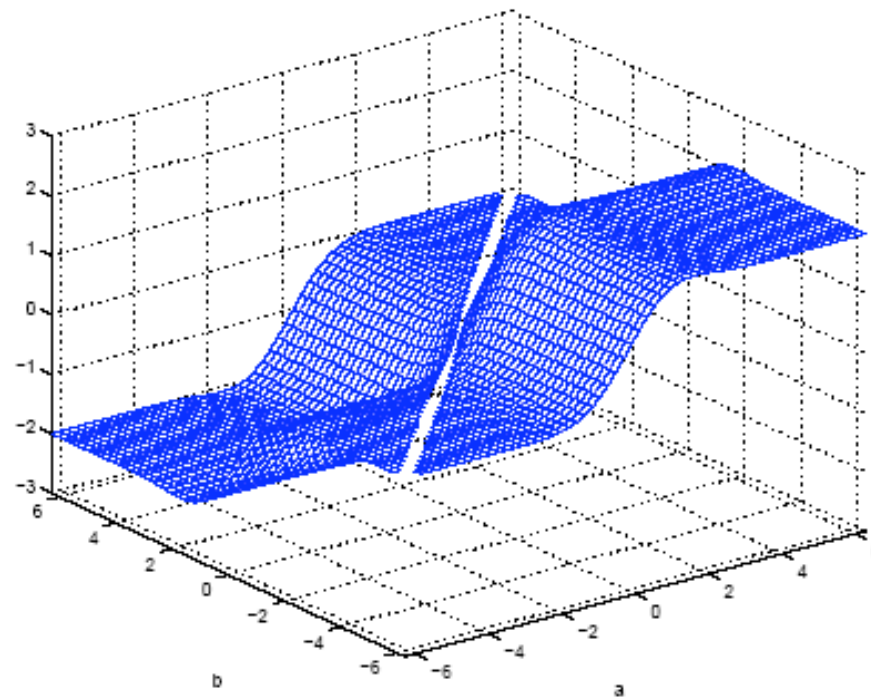
**Gaussian:**  $\varepsilon_{(s,t)} := e^*(s,t) / \text{var}(b)$

$$\varepsilon_{(s,t)} = \varepsilon_{(s-1,t-1)} - [(\beta_{(s,t)})^2 - 1][2\Phi(\beta_{(s,t)}) - 1] \\ - (2/\sqrt{2\pi}) \beta_{(s,t)} \exp(-(\beta_{(s,t)})^2 / 2)$$

$$\varepsilon_{(t,t)} = 0, \quad \varepsilon_{(0,t)} = t$$

# The Best Interval in Gaussian Case

Finding the optimal interval



Plot of the objective function in the Gaussian case with  $T_{(s,t)} = [a, b]$  when  $\sigma_x^2 = 1$ ,  $e_{(s-1,t-1)}^* = 3$ ,  $e_{(s,t-1)}^* = 1$

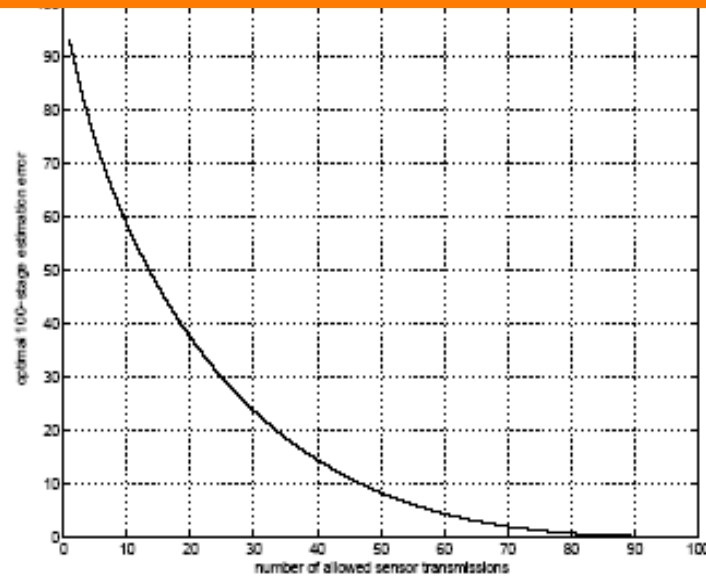
# An Illustrative Example

**Problem:** Given a time-horizon of length  $N=100$ , estimate the state of a zero-mean *i.i.d.* Gaussian process with unit variance

**Design Criterion:** The cumulative estimation error should not exceed 20.

**Solution I:** Make 80 sensor transmissions picked at arbitrary times

**Solution II:** Use the optimal sensor transmission and estimation policies



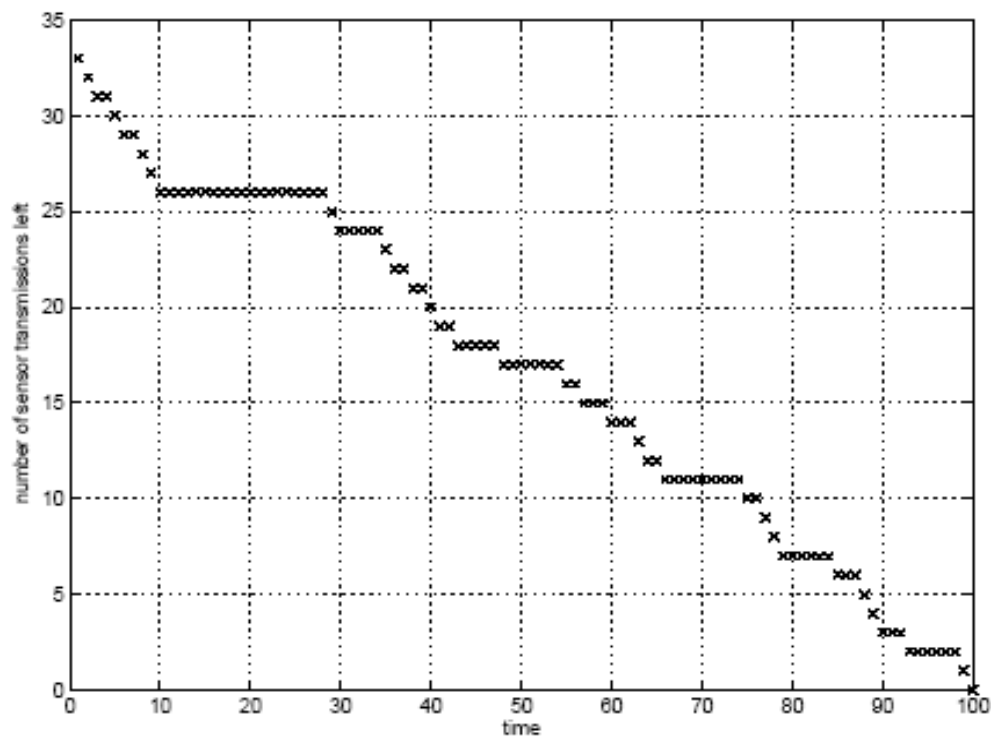
# An Illustrative Example (cont.)

Estimation error of 20 can be achieved with **34 transmissions!**  
This is approximately **58% improvement** --- considerable savings in battery power (if sensor is power-limited) or transmission slots (if the sensor is time-slot limited).

## **Design Verification:**

1. Plot of optimal sample paths of the number of sensor transmissions left, starting with  $(N, M) = (100, 34)$
2. Comparison of the sample path cumulative estimation errors between the optimal and an arbitrary policy

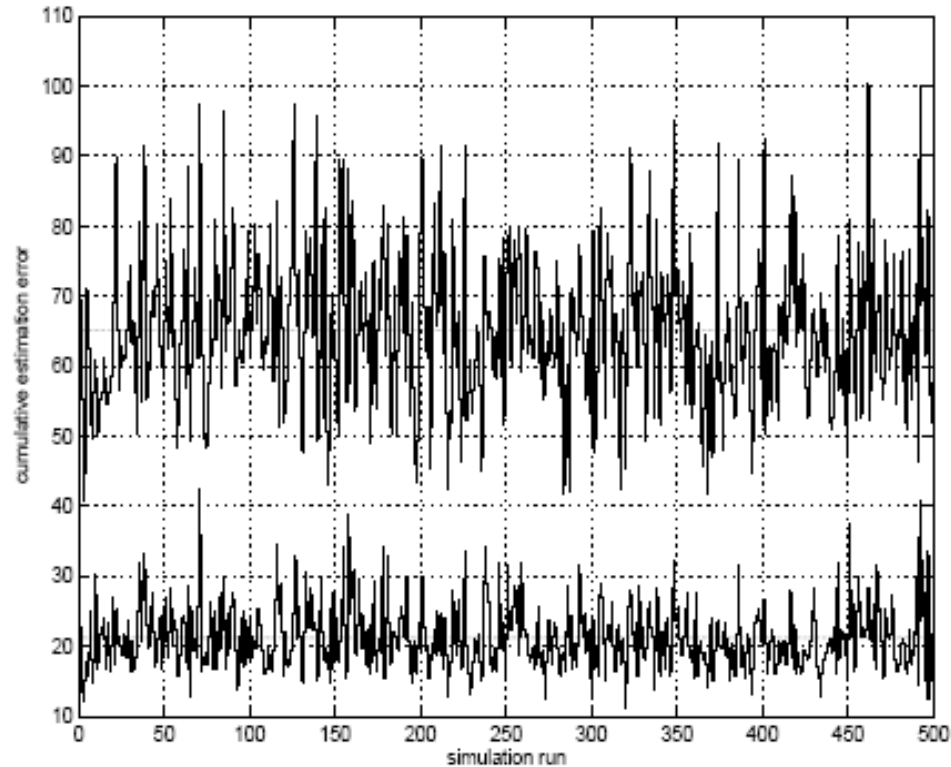
# An Illustrative Example (cont.)



Typical sample path of the number of sensor transmissions left under the optimal transmission policy of the sensor

$$(N, M) = (100, 34)$$

# An Illustrative Example (cont.)



Comparison of the sample path cumulative estimation errors between the optimal and an arbitrary transmission policy

$$(N, M) = (100, 34)$$

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# Source as a Markov Process

$$\mathbf{b}_{k+1} = \mathbf{A} \mathbf{b}_k + \mathbf{w}_k \quad \{\mathbf{w}_k\} \text{ GWN}$$

Optimum sensor policy: keeps track of 3 variables  $(r_k, s_k, t_k)$

$r_k$  : # time units passed since last transmission

At time  $k$  transmit  $\mathbf{b}_k$  if it is in a measurable set  $\mathbb{Y}(r_k, s_k, t_k)$ , otherwise do not

$\mathbb{Y}(r, s, t)$  obtained offline as the minimizer in a recursive equation satisfied by accumulated optimum error,  $e^*(r, s, t)$ , at each  $(r, s, t)$ :

$$e^*(r, s, t) = \min_{\mathbb{Y}(r, s, t)} \left\{ e^*(1, s-1, t-1) \text{Prob}(\mathbf{b}_{N-t} \in \mathbb{Y}) \right. \\ \left. + e^*(r+1, s, t-1) \text{Prob}(\mathbf{b}_{N-t} \notin \mathbb{Y}) \right. \\ \left. + \text{average error at } (r, s, t) \text{ due to decision at } (r, s, t) \right\}$$

# Again an interval solution

$$\forall^c(r,s,t) = [\alpha_{(r,s,t)}, \beta_{(r,s,t)}]$$

$$\beta_{(r,s,t)} = A^r b_{N-t-r} + \sqrt{\{e^*(1,s-1, t-1) - e^*(r+1,s, t-1)\}}$$

$$\alpha_{(r,s,t)} = A^r b_{N-t-r} - \sqrt{\{e^*(1,s-1, t-1) - e^*(r+1,s, t-1)\}}$$

$$\varepsilon_{(r,s,t)} := e^*(r,s,t) / \sum_{k=1}^r A^{2(k-1)} \text{var}(b_0)$$

$$\varepsilon_{(r,s,t)} = \varepsilon_{(1,s-1, t-1)} - [(\nu_{(r,s,t)})^2 - 1][2\Phi(\nu_{(r,s,t)}) - 1] \\ - (2/\sqrt{2\pi}) \nu_{(r,s,t)} \exp(-(\nu_{(r,s,t)})^2 / 2)$$

$$\nu_{(r,s,t)} := \sqrt{\{e^*(1,s-1, t-1) - e^*(r+1,s, t-1)\}}$$

# Recap: Smart Sensing

## Input parameters:

1. **Planning horizon**: Desired length of time the sensor will be in operation (**N**)
2. **Battery size**: Size of the power source installed into the sensor (**M**)
3. **Process model (data)**: This is application specific, and can be gathered from a variety of sources including, measurement/historic data, modeling, etc. (**Source Statistics**)
4. **Performance criterion**: (**Distortion Measure**)

# Smart Sensing

## How?

- Given the input parameters and the channel compute the optimum transmission and decoding policies for the wireless sensor & the receiver
- The computation is “power aware” (M), and “planning-horizon aware” (N)
- Minimal online computational complexity (lookup table, binary comparisons,...)
- Offline computational complexity will depend on the source and channel models

# Outline

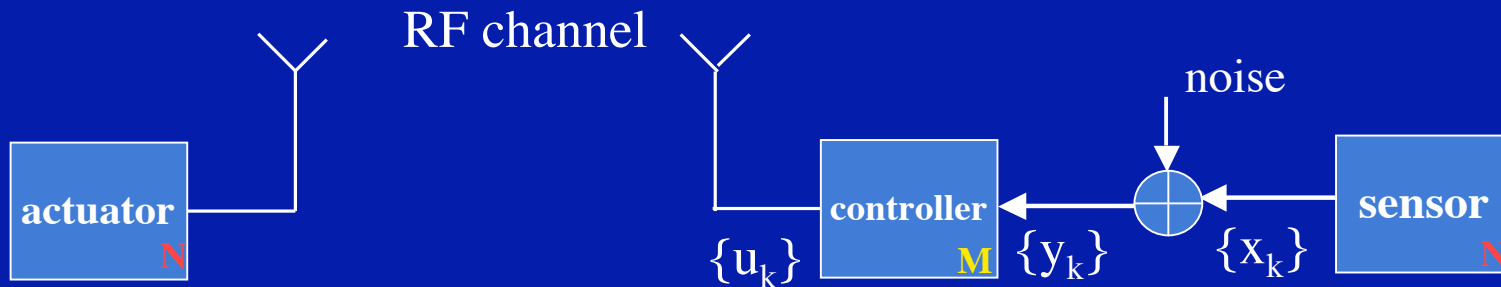
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# PROBLEM CLASS II

## OPTIMAL CONTROL

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# Control over a Limited-Use Channel



$$x_{k+1} = Ax_k + u_k + w_k \quad x_k \in R, u_k \in R, y \in R$$

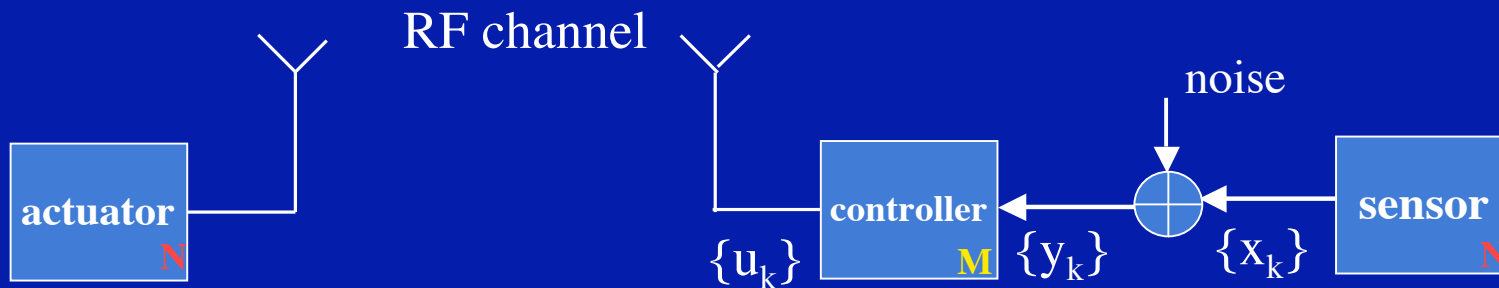
$$y_k = x_k + v_k \quad M < N$$

$$u_k = \mu_k(I_k), \quad I_k := \{y_{[0,k]}, u_{[0,k-1]}\}, \quad \mu_k \rightarrow R \text{ only } M \text{ times}$$

*Given a horizon of  $N$  units, and with control allowed to transmit for only  $M < N$  times, what is the minimum attainable value of a performance index  $J$ , and a corresponding controller?*

$$J = E\{ (x_N)^2 + \sum_0^{N-1} (x_k)^2 \}$$

## Control over a Limited-Use Channel



“Open-loop controller schedule”: Determine *a priori*  $M$  time slots (out of  $N$ ) when to apply control, and find the best such  $M$  slots as a result of combinatorial optimization.

$$\begin{aligned} \text{Result: } u_k &= -A E[x_k | I_k] && \text{for } 0 \leq k \leq M-1 \\ &= 0 && \text{for } M \leq k \leq N \end{aligned}$$

# Closed-Loop Controller Schedule

Scheduling of control is on-line/*a posteriori*

- $s$  : # control actions left (a total of  $M$ )
- $t$  : # decision instances left (a total of  $N$ )
- Given  $(M, N)$ , in retrograde time,  $t : 1 \rightarrow N$   
$$\max\{0, M-(N-t)\} \leq s \leq \min\{t, M\}$$
- With  $t$  fixed, the maximum interval for  $t$  is  $[0, t]$
- Approach is DP, moving forward in  $t$   
 $(s, t)$  can lead to either  $(s-1, t-1)$  or  $(s, t-1)$

# Closed-Loop Controller Schedule (continued)

## Online Computation

- For every  $k$ ,  $x_k - E[x_k | I_k]$  is independent of control
- If, at  $k$ , decision is to control, the optimum choice is

$$u_k = -A E[x_k | I_k]$$

where  $E[x_k | I_k]$  is generated by Kalman filter.

- Decision is to control, if  $|E[x_k | I_k]|$  exceeds a certain threshold,  $\tau(s_k, t_k)$

$$\implies s_{k+1} = s_k - 1; \quad t_{k+1} = t_k - 1$$

# Closed-Loop Controller Schedule (continued)

## Offline Computation

- $\tau(s_k, t_k)$  is a function of  $N, s_k, t_k, \sigma_{k|k-1}^2$
- Compute optimum cost-to-go's at  $(s,t)$ , based on whether a control was applied,  $J_{(s,t)}^{(1)}$ , or not,  $J_{(s,t)}^{(0)}$
- $\Delta_{(s,t)} := J_{(s,t)}^{(0)} - J_{(s,t)}^{(1)} = 0$  determines the threshold (it has two roots -- one positive and one negative)

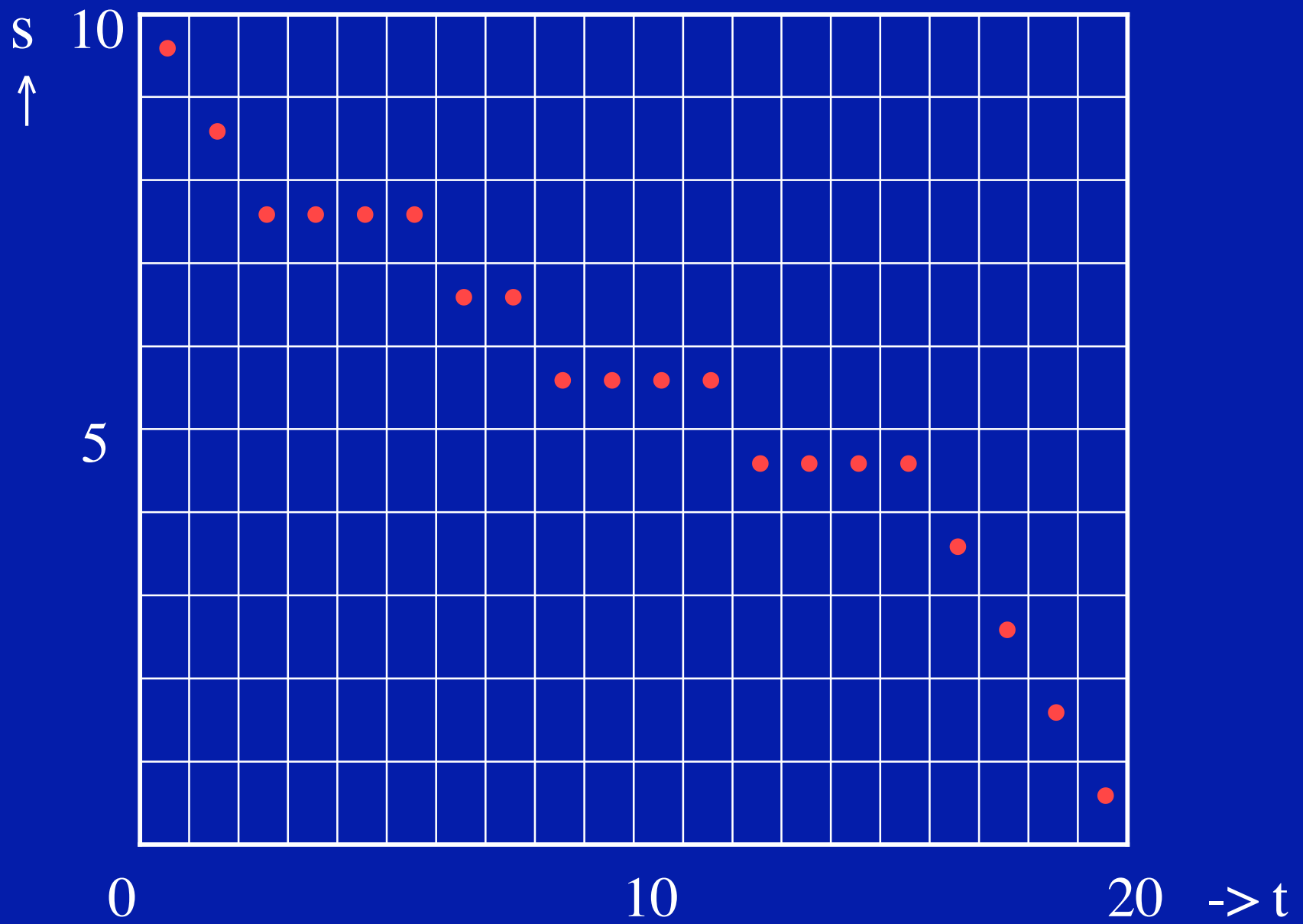
$$\Delta_{(s,t)}(\tau(s, t)) = \Delta_{(s,t)}(-\tau(s, t)) = 0$$

# Numerical Solutions

**Numerical integration was used to compute the recursions  $\Delta_{(s,t)}$ , which led to thresholds  $\tau(s, t)$**

**Implemented the optimal control with M actions for an N-stage problem (N=20). Computed  $J_{(M,N)}^*$  based on sample paths, and for different M values**

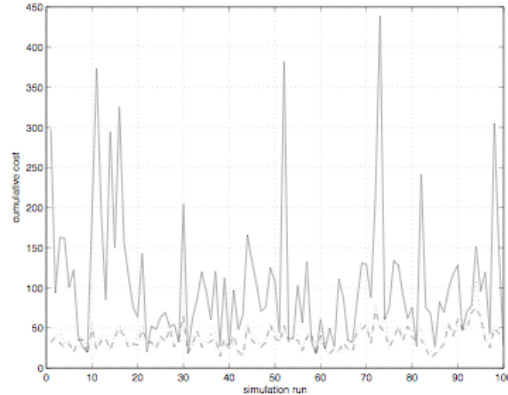
Times of control action  $\Rightarrow$



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# Numerical Solutions (continued)

$$N = 20, A = 1, \sigma_w^2 = \sigma_v^2 = 1$$



Sample path N-stage costs, for  $M = 1, 10, 20$   
100 simulation runs

M	$J^*_{(M,N)}$	%
1	96.4266	203.9327
2	68.1907	114.9343
3	47.2060	48.7914
4	44.0160	38.7366
5	39.8642	25.6503
6	37.1557	17.1132
7	35.6168	12.2627
8	34.1551	7.6555
9	33.6935	6.2005
10	33.6913	6.1936

M	$J^*_{(M,N)}$	%
11	33.2445	4.7853
12	32.9262	3.7820
13	32.8267	3.4684
14	32.4936	2.3249
15	32.1082	1.2037
16	31.9824	0.8072
17	31.8822	0.4914
18	31.8417	0.3637
19	31.7337	0.0233
20	31.7263	0

# Outline

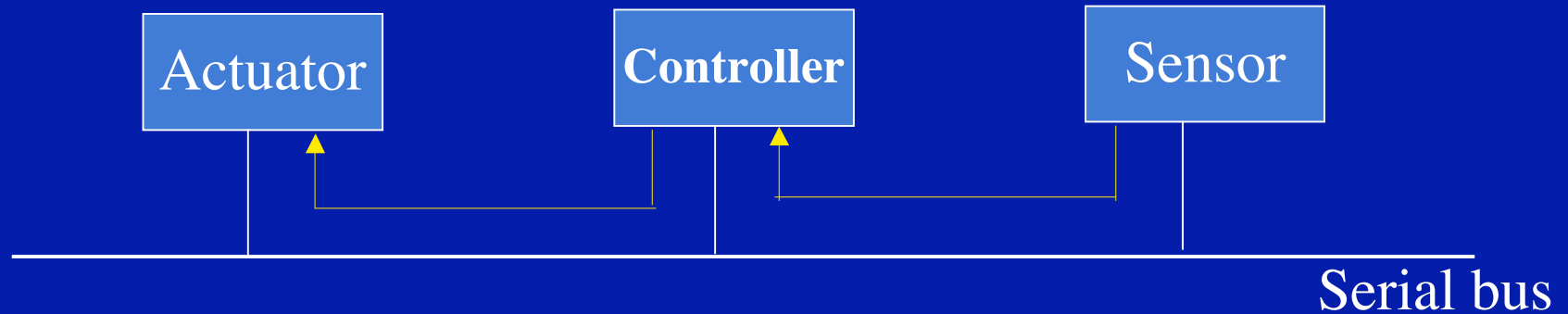
- Networks & Control
- Joint sensor-controller design
  - What/when/how to transmit & control
- Estimation with “Power-Limited Communication”
- Control with “Power-Limited Communication”
- **When to measure & when to control**
- Other issues / Conclusions

# PROBLEM CLASS III

## OPTIMAL MEASUREMENT AND CONTROL

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# Scheduled Measurements & Controls



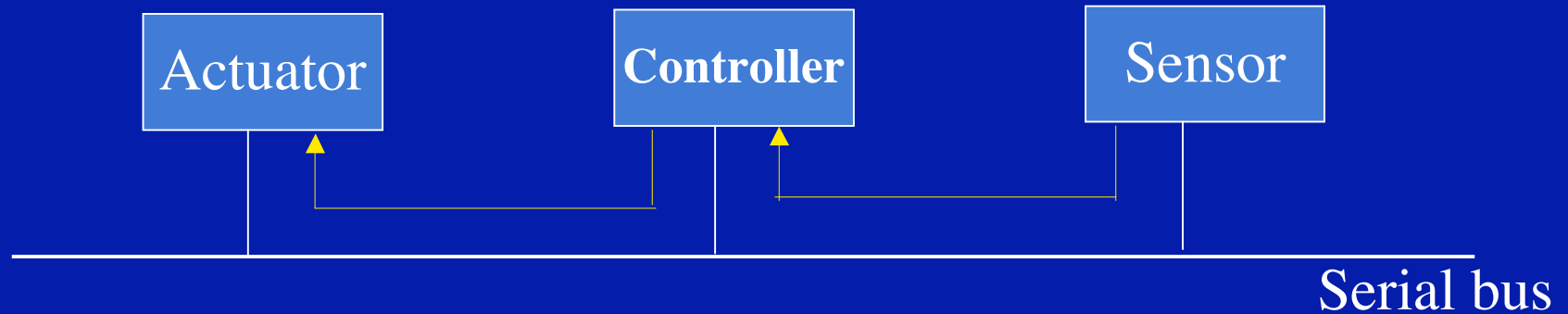
$$\mathbf{x}_{k+1} = \mathbf{A} \mathbf{x}_k + \mathbf{a}_k \mathbf{u}_k + \mathbf{w}_k, \quad k = 0, \dots, N-1$$

$\mathbf{a}_k$  is 0 or 1

$$\mathbf{y}_k = (1 - \mathbf{a}_k) \mathbf{x}_k, \quad k = 0, \dots, N-1$$

measurement

# Scheduled Measurements & Controls

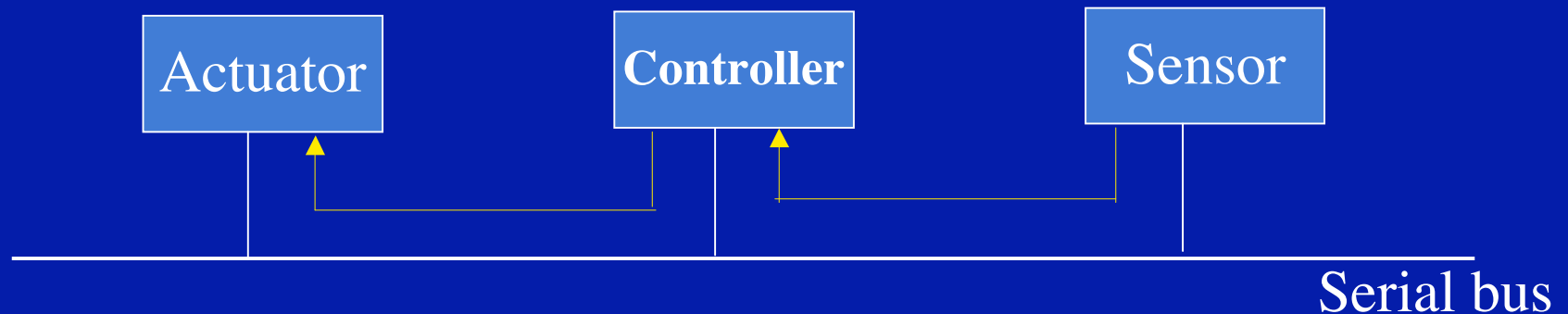


$$\mathbf{x}_{k+1} = \mathbf{A} \mathbf{x}_k + \mathbf{a}_k \mathbf{u}_k + \mathbf{w}_k, \quad k = 0, \dots, N-1$$

**Policies:**  $(\alpha_k(\mathbf{I}_k), \mu_k(\mathbf{I}_k)) \rightarrow (0, 0) \text{ or } (1, \mathbf{u}_k)$

$$\mathbf{I}_k = \{ \mathbf{y}_{[0, k-1]}, \mathbf{u}_{[0, k-1]} \}, \quad k=1, \dots, N-1$$

# Scheduled Measurements & Controls



$$\mathbf{x}_{k+1} = \mathbf{A} \mathbf{x}_k + \mathbf{a}_k \mathbf{u}_k + \mathbf{w}_k, \quad k = 0, \dots, N-1$$

**Find**  $\pi = \{(\alpha_k(\mathbf{I}_k), \mu_k(\mathbf{I}_k)), k=1, \dots, N-1\}$

minimizing  $J_\pi = \mathbf{E}\{x_N^2 + \sum_0^{N-1} x_k^2\}$

# Optimal open-loop scheduled policy:

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Optimal open-loop scheduled policy:

**M C M C M C M C M C M C**

Optimal open-loop scheduled policy:

**M C M C M C M C M C M C M C**

$$u_k = -A^2 x_k$$

Optimum OL average cost (infinite horizon)

$$J_{OL}^* = (1 + A^2 + (1/2) A^4) \text{var}(w)$$

## Optimal closed-loop scheduled policy:

A threshold policy (as in the previous case)

Propagate cost-to-go's  $J_k^{(0)}$  and  $J_k^{(1)}$

This leads to offline computation of the difference  $\Delta_k(\tau)$ , whose positive root is the optimal threshold at time  $k$ .

# Optimal closed-loop scheduled policy (continued)

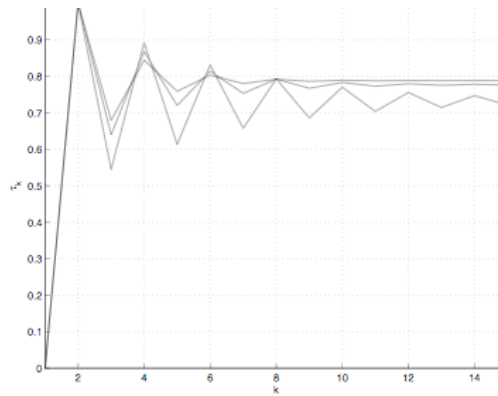
## Online implementation:

At  $k=0$ , start by measuring  $(M)$ . For other  $k$ :

$$\begin{aligned} (\mathbf{a}_{k+1}, \mathbf{u}_{k+1}) &= (0,0) && \text{if } a_k = 0, |x_k| \leq \tau_k \\ &= (1, -A^2 x_k) && \text{if } a_k = 0, |x_k| > \tau_k \\ &= (0,0) && \text{if } a_k = 1 \end{aligned}$$

# Numerical Study

Convergence of  $\tau_k$  as  $k \rightarrow \infty$



$\text{var}(w) = 1, A=1, \tau_k$  converges: **0.7767**

**Does not show much sensitivity to A**

**Rate is sample-path dependent**

# Other Settings

- Noisy measurements (KF with uncertain observations)
- Multi-dimensional case (conceptually similar)
- Cost on control (no longer linear)
- Unreliable/failure prone links for sensing and control transmission
- Multiple agents with infrequent exchange of information and decision
- Nonlinear systems

# Recap

- Networks & Control
- Joint sensor-controller design
  - What//when/how to transmit & control
- Estimation with “Power-Limited Communication”
- Control with “Power-Limited Communication”
- When to measure & when to control

**Literature:** OCI-TB CDC'05; ACC'06 (2)