Control of Congestion in High-Speed Networks

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OUTLINE

1. High speed networks with heterogenous traffic; types of sources and classes of service; QoS; ATM Networks vs Internet
2. Flow control. Best effort traffic.
   Control-theoretic issues.
   Regulation of flows on high speed networks
3. Information flow
   - ◇ RM Cell transmission (ATM)
   - ◇ AQM or "marking" schemes (TCP/IP)
4. General network model
   - ◇ Fluid approximation
   - ◇ Bottleneck links
   - ◇ Role of routers (switches)
     - ○ Explicit rate scheme (ABR/ATM)
     - ○ Marking schemes (ECN, RED, REM)
5. Link-level explicit rate congestion control
   - ◇ A robust adaptive algorithm
   - ◇ Stability (unknown delays and # users)
   - ◇ Examples and simulations
6. Network-level implementation
   - ◇ A hybrid control system
   - ◇ Stability
7. Extension to marking-based schemes
8. Other issues .... Recap

High-speed network (B-ISDN):

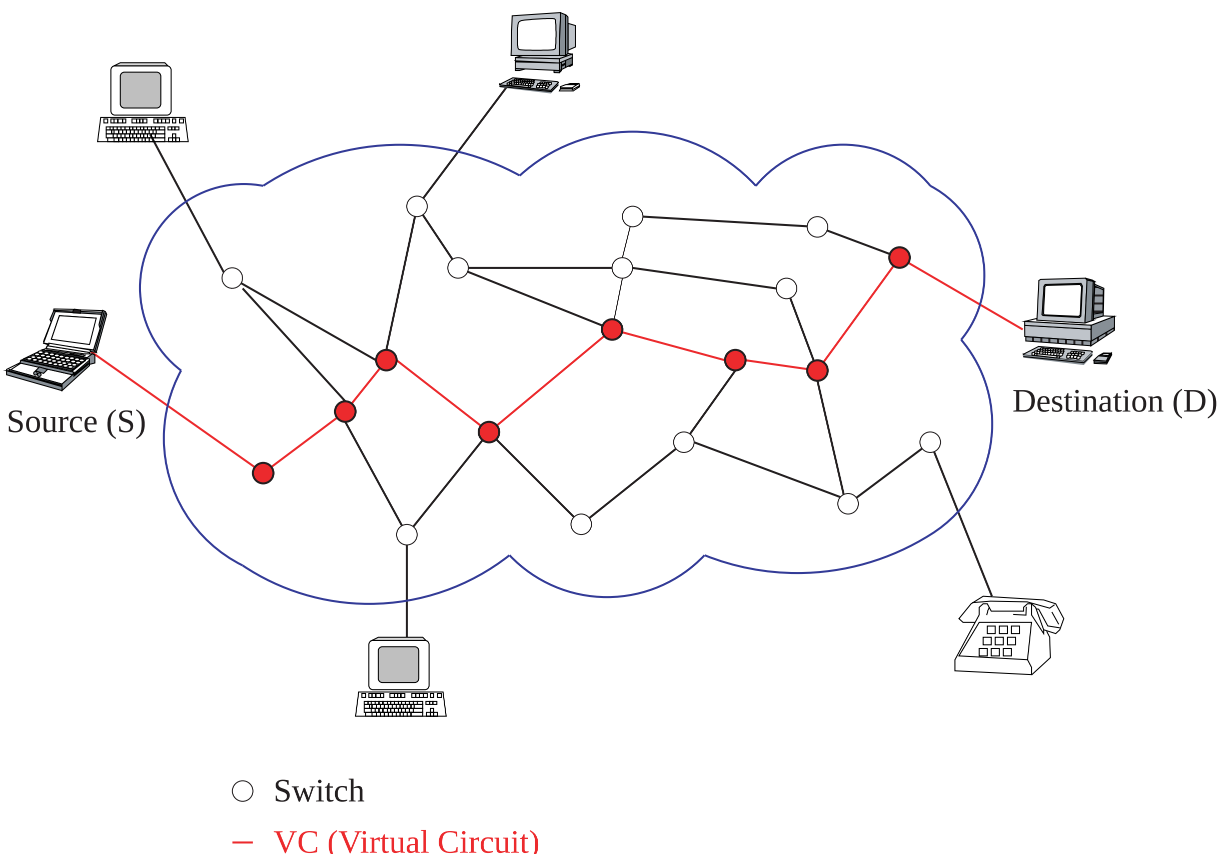
Collection of nodes connected by links of varying capacity (bits-per-second)

Different types of sources (and service)

- **Real-time: Voice, video (guaranteed QoS)**

  – upper bounds on packet loss probability, delay, etc.

- **Best-effort: Email, Web browsing (ABR)**



Issues / Services:

- **Admission control**

  How many sources can be admitted subject to some QoS requirements (upper bound on fraction of packets lost, maximum delay, etc.)

- **Routing**

  ◇ **Datagram Service (Internet):**

  Each packet routed independently of others

  Routes chosen to minimize average packet delay

  ◇ **Virtual Circuit Service (ATM network):**

  Each packet from a source follows the same path

  An effective BW associated with each source

  A call admitted if EBW $<$ BW available on a link

  Routes chosen to maximize # sources admitted

- **Congestion Control**

  Efficient use of available capacity among best-effort sources; fair allocation of resources

- **Scheduling**

  Packets are indivisible entities (of varying size)

  Impact on QoS

Congestion control for "best-effort" sources

- Reactive flow control and congestion control used to regulate transmission of information from a source to a destination : Window flow control ; Credit-based flow control ; Rate-based flow control
- Examples:
  - ◇ TCP/IP congestion control in the Internet
    Sources make decisions
  - ◇ Available Bit Rate (ABR) service-type in ATM.
    Network (intermediate nodes) makes decisions
- Best effort (ABR) vs guar. services (CBR, VBR)
- Design objectives: "The primary role of the parameters and procedures used for traffic control and congestion control is to protect the network and the-end-system in order to achieve network performance objectives. An additional role is to optimize the use of network resources... The design of ATM layer traffic and congestion controls should minimize network and end-system complexity while maximizing network utilization".
  (The ATM Forum Technical Committee, *Traffic Management Specification*, Version 4.0, ATM Forum/95-0013R8, October 1995.)
- Advantages: Efficient use of network resources, high throughput switches, guarantee of QoS .....
  – high-speed backbone for existing networks

Objectives

1. **Efficient use of network resources while meeting the QoS requirements:**
   - **minimal cell loss rate**
   - **average delay of all "connections"**
   - **meeting MCRs and not exceeding PCRs**
2. **Maintain "fairness" among all sources sharing a link**

Congestion control

Adjust the rate of flow into the network by each active source, based on feedback received on the congestion state of the network

Challenges:

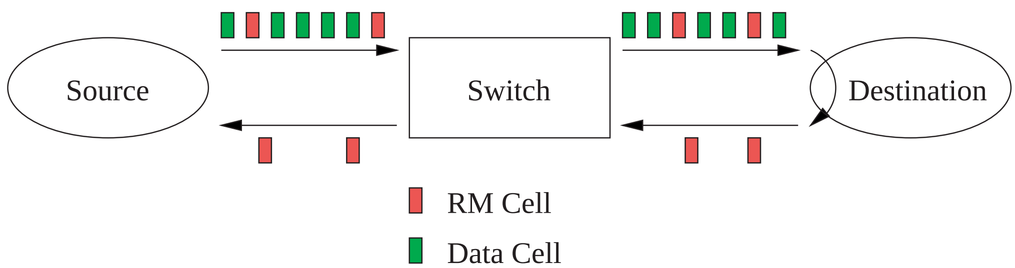
- **How to handle unpredictable delay (propagation, queueing, processing)**
- **Traffic characteristics: connection lifetime; numbers of active and controlled sources on each link not known**
- **Ease of implementation**

3. Information Flow

BW availability, level of congestion, impending congestion

◇ **In ATM Networks: through RM Cells**

- **53 byte long cells**
- **Source transmits at ACR (Allowed Cell Rate)**
- **Initially ACR ← ICR**
- **S generates RM cells every 32 data cells**
- **RM cells are sent back to S by Destination**



RM cell structure

Header	CI	NI	ER (Explicit Rate)	CCR (Current Cell Rate)	Reserved

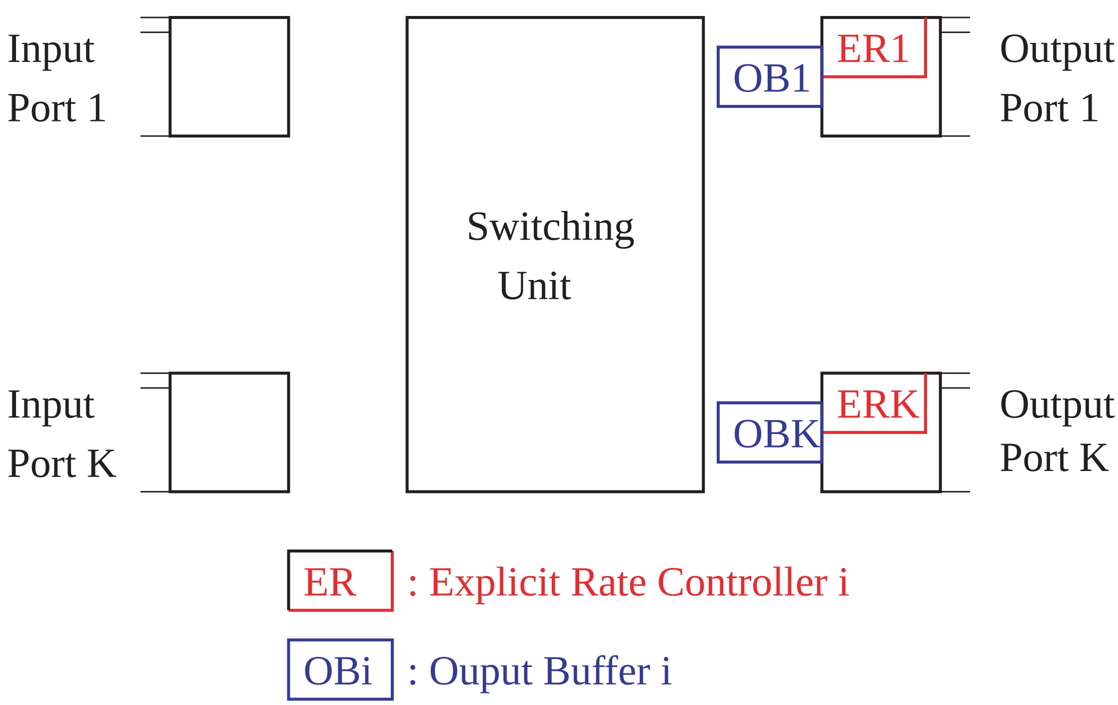
CI (Congestion Indication) Bit
NI (No Increase) Bit

Rate modification phase :

- **S sets ER ← PCR, CCR ← ACR**
- **Switches along VC: ER ←** min{**ER, CR**}
- **ER controller at switch determines CR**

ATM Switch Architecture

- Equal number of (K) input and output ports
- Asynchronous cell arrival
  Synchronous switching
- Commercial switches use Output Queueing
- Queueing is NOT per VC



Information flow

BW availability, level of congestion, impending congestion

◇ **Internet:** **No explicit feedback from routers**

TCP/IP : end-to-end flow control protocol

- **uses window-based congestion control**
- **window sizes adjusted using packet losses and timeouts as congestion indicators**
- **does not always perform well**

  (e.g. in wireless channels)

Remedy: AQM or "marking" systems.

Packets marked at routers probabilistically, based on queue length

4. General Network Model

Graph of communication links $(\mathcal{L}, \mathcal{S})$

$\mathcal{L} := \{1, \ldots, L\}$ **: set of links**

$\mathcal{S} := \{1, \ldots, S\}$ **: set of connections using** $\mathcal{L}$

$\mathcal{L}_s \subset \mathcal{L}$**: set of links used by connection** s

$\mathcal{S}_l \subset \mathcal{S}$**: set of connections using link** l

C_l **: Capacity of link** l $\quad C := \{C_l | l \in \mathcal{L}\}$

r_s**: flow rate of connection** s**;** $\quad x_s := r_s - \mathrm{MR}_s$

F_l**: aggregate flow on link** l

$$F_l = \sum_{s \in \mathcal{S}_l} r_s = \sum_{s \in \mathcal{S}_l} x_s + G_l \leq C_l$$

FAIRNESS:

- x **is max-min fair if** x_s **cannot be increased without decreasing** $x_{s'}$ **for some connection** s' **for which** $x_{s'} \leq x_s$
- x **is proportionally fair if for any other feasible vector** x'**,**

$$\sum_{s \in \mathcal{S}} \frac{x'_s - x_s}{x_s} \leq 0$$

BOTTLENECK LINK:

A link l is a *bottleneck link* with respect to x for a connection s crossing l if $C_l = F_l$ and $x_s \geq x_{s'}$ for all connections s' crossing link l.

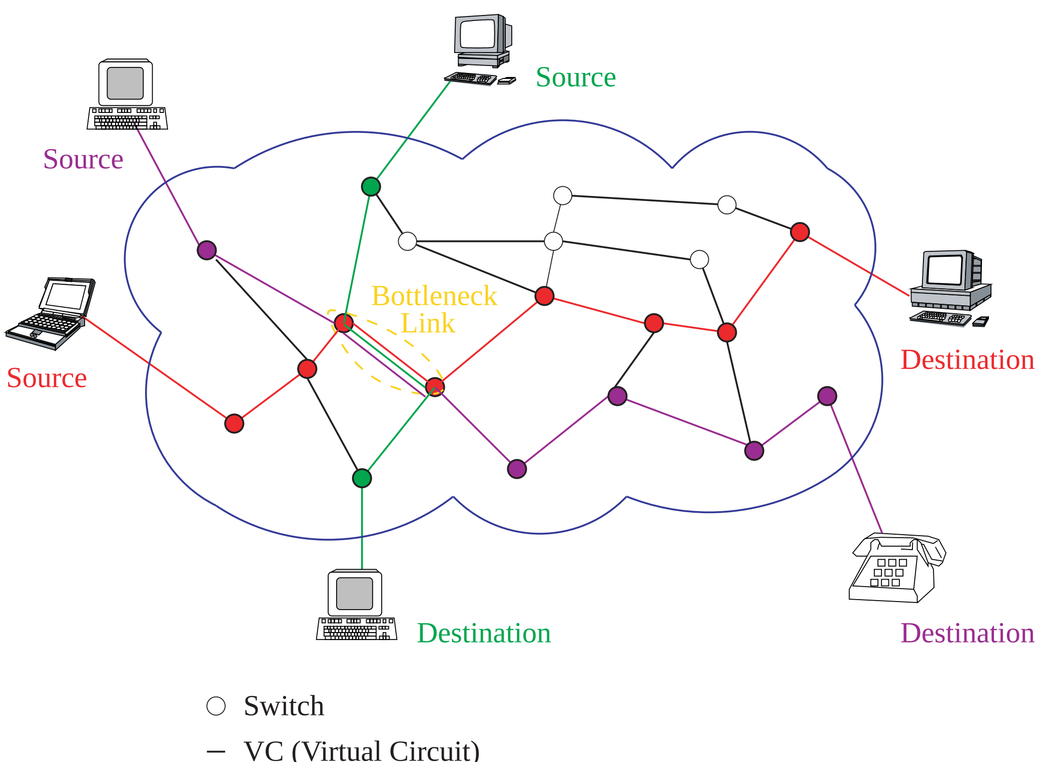
Max-min fair allocation $\Rightarrow$

each connection has a bottleneck link (BL)

$\mathcal{B}_l \subset \mathcal{S}$: connections that have l as a BL

In a max-min fair allocation, $p, q \in \mathcal{B}_l \Rightarrow x_p = x_q$

$$r_s^* = \mathrm{MR}_s + \frac{C_l - (u_l + G_l)}{M_{l,l}(\infty)} \qquad \forall s \in \mathcal{B}_l$$



Can max-min fair allocation r^* be achieved while

- **maintaining a high utilization of the resources (link capacities)**

and

- **maintaining a low loss rate for all connections**

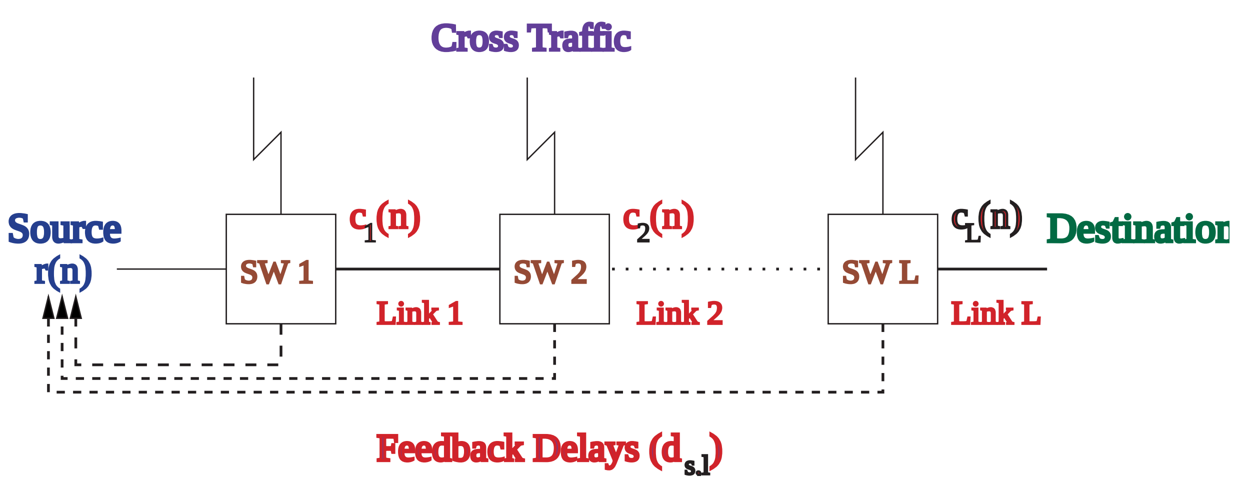
Regulate queue length at routers around a target level $\Rightarrow \quad q_l \sim Q_l^* < Q_l \quad$ (*size of link buffer*)

Goal of `Congestion Control Algorithm :`

$$q_l \to Q_l^*\,, \quad \forall\, l \in \mathcal{L} \quad \textbf{and} \quad r_s \to r_s^*\,, \quad \forall\, s \in \mathcal{S}$$

$$q_l(n+1) = \max\left\{0, \min\left\{Q_l, q_l(n) + F_l(n) - C_l\right\}\right\}$$

$$r_s(n) = \mathrm{MR}_s + \min_{l \in \mathcal{L}_s} \mathrm{ER}_l(n - d_{s,l})$$



ATM/ABR congestion control

Control challenge: **Find a DECENTRALIZED update scheme for $\mathrm{ER}_l(n)$, $l \in \mathcal{L}$, which does not require per-flow information**

$\Rightarrow$ $\mathrm{ER}_l(n - d_{s,l}) = \mu_{s,l}(F_l(n), q_l(n))$

Marking-based congestion control

→ **On link l, mark packets with prob.** $1 - e^{-\lambda_l(n)}$

→ **Source s measures the fraction $f_s(n)$ of unmarked packets in time slot n:**

$$f_s(n) = e^{-\sum_{l \in \mathcal{L}_s} \lambda_l(n - d_{s,l})}$$

→ **Source s can estimate $\sum_{l \in \mathcal{L}_s} \lambda_l(n - d_{s,l})$ as**

$$\sum_{l \in \mathcal{L}_s} \lambda_l(n - d_{s,l}) = -\ln f_s(n)$$

→ **Rate r_s of source s can be updated as**

$$r_s(n) = \mathrm{MR}_s + g\left(\sum_{l \in \mathcal{L}_s} \lambda_l(n - d_{s,l})\right)$$

$g(\cdot)$ **: monotonically decreasing fn – control**

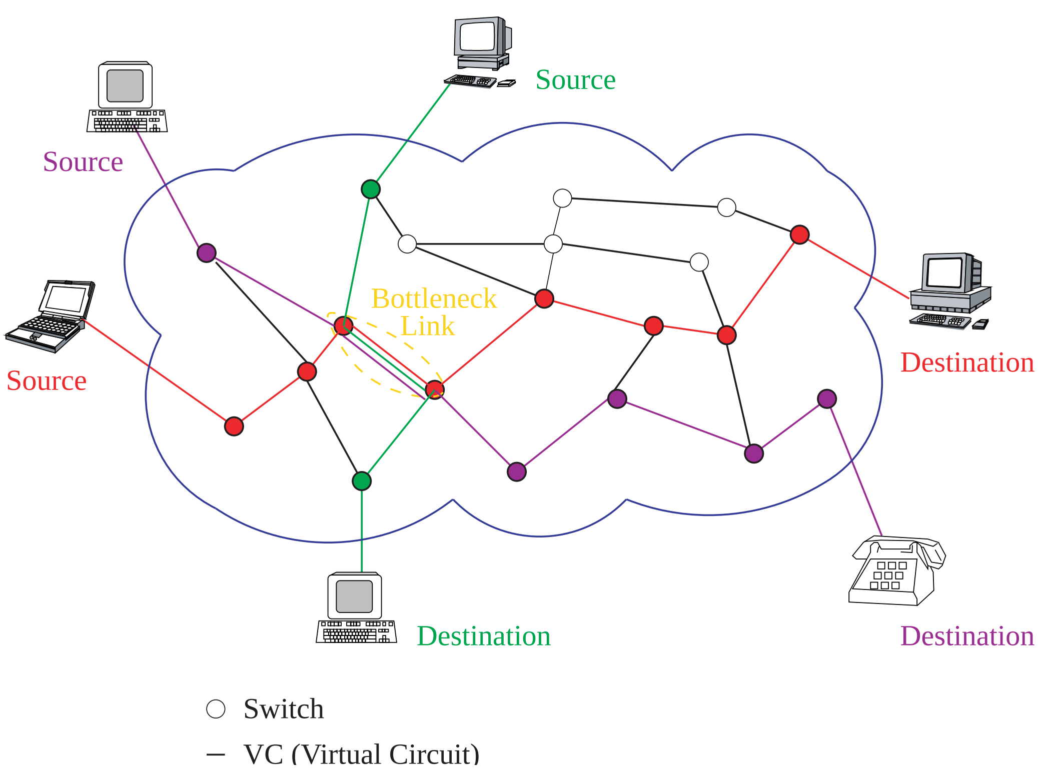
5. A link-level ER Congestion Control Algorithm

Assumptions:

1. **Single bottleneck link** l
2. $M_l = \text{card}(\mathcal{S}_l)$ **does not change with time**
3. $M_{l,l}(n) = \text{card}(\mathcal{B}_l(n))$ **is time-invariant**

   $\Rightarrow M_{l,l}(\infty)$ **not known to the switch**
4. $u_l(n)$ **varies slowly**

   $\Rightarrow u_l(\infty)$ **not known to the switch**
5. $0 \leq d_{1,l} \leq \cdots \leq d_{M_{l,l}(n),l} \leq b_l$
6. $\text{ER}(\cdot)$ **is updated every** $(b_l + 1)$ **time units**



Queue dynamics

$$q_l(n+1) = \max\{0, \min\{Q_l, q_l(n) + F_l(n) - C_l\}\}$$

$\Rightarrow$

$$q_l(n+1) = \max\Big\{0, \min\Big\{Q_l, q_l(n) + \sum_{k=0}^{b_l} m_{k,l}(n)\mathrm{ER}_l(n-k) + u_l(n) - (C_l - G_l)\Big\}\Big\}$$

$q_l^d(n),\ \mathrm{ER}_l^d(n),\ F_l^d(n)$**: sampled values (at** $n(b_l+1)$**)**

ER update algorithm:

$$\mathrm{ER}_l^d(n) = \max\Big\{0, \min\Big\{C_l, \mathrm{ER}_l^d(n-1) - \alpha_l\big(F_l^d(n) - C_l\big) - \beta_l\big(q_l^d(n) - Q_l^*\big)\Big\}\Big\}, \quad \mathrm{ER}_l^d(0) = C_l$$

If stable, then

$$\mathrm{ER}_l^d(\infty) = \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)}$$

$$r_s(\infty) = \mathrm{MR}_s + \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)} \qquad \textbf{max-min fair}$$

$$q_l^d \to q_l(\infty) = Q_l^*$$

(α_l, β_l) **can be picked to make the 2-dim system (queue dynamics and controller dynamics) g.a.s.**

An alternative optimization approach

Minimize over

$$r_s(n) = \mathrm{MR}_s + \mathrm{ER}_s(n - d_{s,l})\,, \quad s \in \mathcal{B}_l,$$

the "team" objective function:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \Big\{ \big[q_l(n) - Q_l^*\big]^2$$

$$+ \sum_{s\in\mathcal{B}_l} \big[\mathrm{ER}_s(n-d_{s,l}) - \nu_{s,l}\big(C_l - G_l - u_l(n)\big)\big]^2 \Big\}$$

$\nu_{s,l}$ **: allocation parameter**

Max-min fair : $\nu_{s,l} = 1/\mathrm{card}(\mathcal{B}_l)$

$\Rightarrow$ **A decentralized optimal control problem.**

A viable approach: certainty equivalence

Analysis

$$x_0(n) := q_l^d(n) - Q_l^* \,,$$

$$x_1(n) := \mathrm{ER}_l^d(n-1) - \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)}$$

$$\Rightarrow \; x(n+1) = \mathrm{sat}\big(A_{\alpha_l,\beta_l} x(n)\big)$$

$$1 \le \gamma_0 \le \bar{\gamma}_0 =: \sum_{k=0}^{b_l} (b_l + 1 - k) m_{k,l}$$

$$0 \le \gamma_1 \le \bar{\gamma}_1 =: \sum_{k=0}^{b_l} k m_{k,l}$$

$$A_{\alpha_l,\beta_l} = \begin{pmatrix} 1 - \beta_l \gamma_0 & \gamma_0(1 - \alpha M_{l,l}) + \gamma_1 \\ -\beta_l & 1 - \alpha M_{l,l} \end{pmatrix}$$

The system (origin) is l.a.s. if

$$0 < \alpha_l < \frac{1}{M_{l,l}(\infty)} \qquad \texttt{AND}$$

$$0 < \beta_l < \min \left\{ \frac{1}{\gamma_0}, \frac{\alpha_l M_{l,l}(\infty)}{\gamma_1} \right\}$$

Global Asymptotic Stability

Conditions:

$$\max(Q_l - Q_l^*, Q_l^*) > p_{\max} \min\left(\frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)}, \left(C_l - \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)}\right)\right)$$

$$(Q_l - Q_l^*)Q_l^* > \left(C_l - \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)}\right) \cdot \frac{C_l - (u_l(\infty) + G_l)}{M_{l,l}(\infty)} p_{\max}$$

$$p_{\max} = f_l(\alpha_l, \beta_l, M_{l,l}, \gamma_0, \gamma_1) \geq 5 + (4/\sqrt{3})$$

The system is g.a.s. if, in addition to above,

$$0 < \alpha_l < \frac{2}{3\bar{M}_{l,l}} \qquad \texttt{AND}$$

$$0 < \beta_l < \alpha \min\left\{\frac{1}{b_l + 1}, \frac{\alpha_l(C_l - (u_l(\infty) + G_l))}{Q_l - Q_l^*}, \frac{\alpha_l[(M_{l,l} - 1)C_l + (u_l(\infty) + G_l)]}{Q_l^*}\right\}$$

$Q_l^* = Q_l/2 \;\Rightarrow\;$ **for** $M_{l,l}(\infty)$ **moderately large,**

$$0 < \beta_l < \alpha \min\left\{\frac{1}{b_l + 1}, \frac{2\alpha_l(C_l - (u_l(\infty) + G_l))}{Q_l}\right\}$$

Simulation : fluid level

Parameters:

$\mathrm{MR}_s = 100, \quad b = 5, \quad u = 400, \quad Q^* = 1500,$

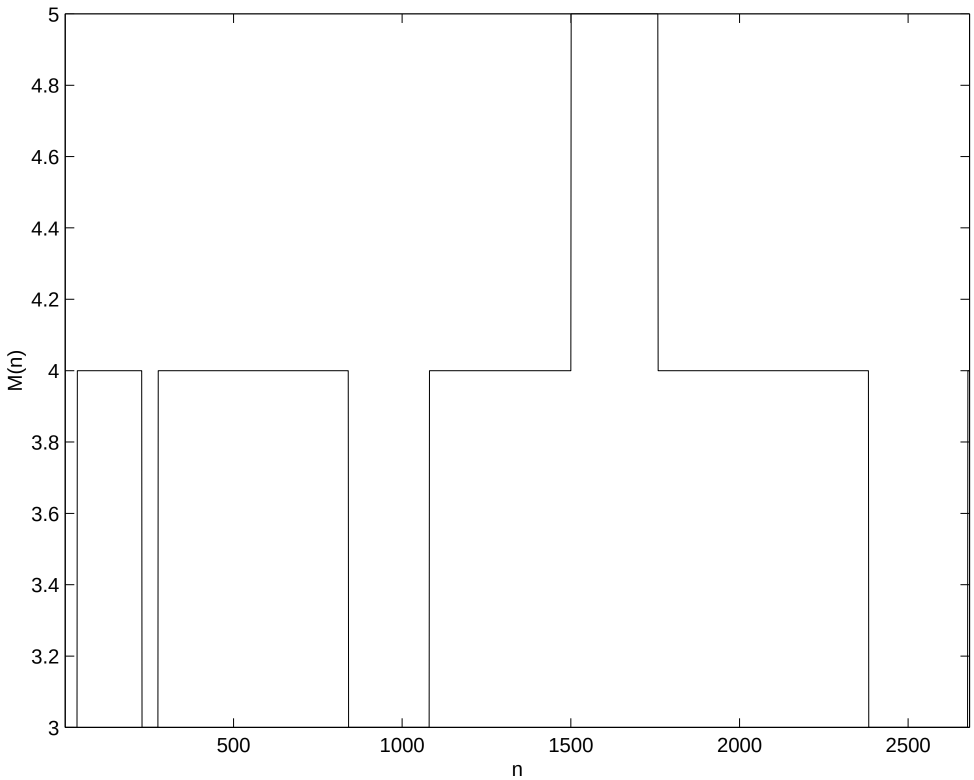
$Q = 3000, \quad C = 1500$

$\alpha = 0.1, \beta = 0.01$

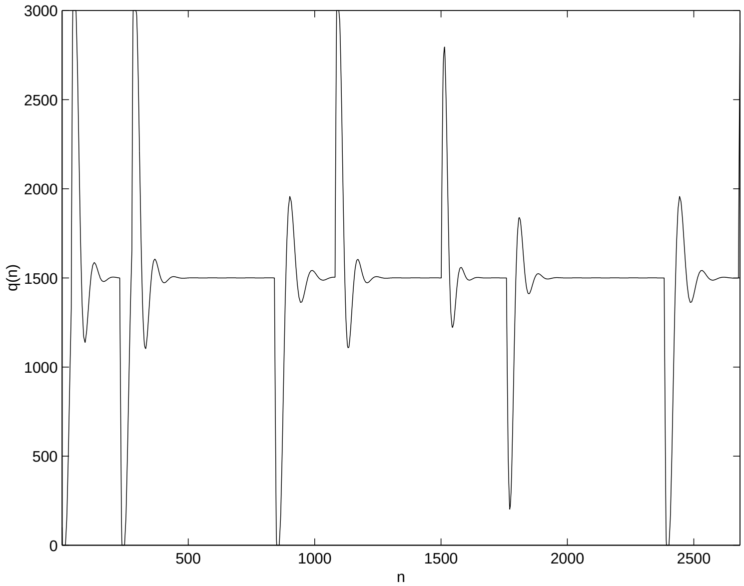
$M(n)$: random walk with boundaries at 1 and $M_{\max}$

–updated every $T(\eta)$ time units (exp rv w mean 200)

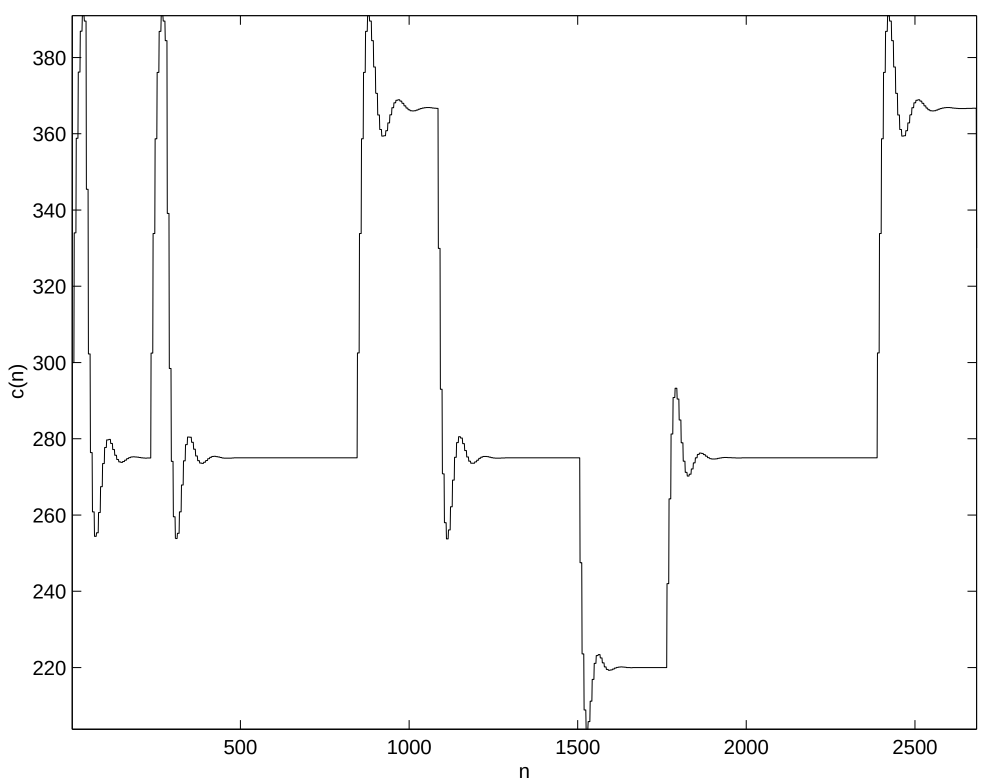
Sample-path behavior of # BN connections



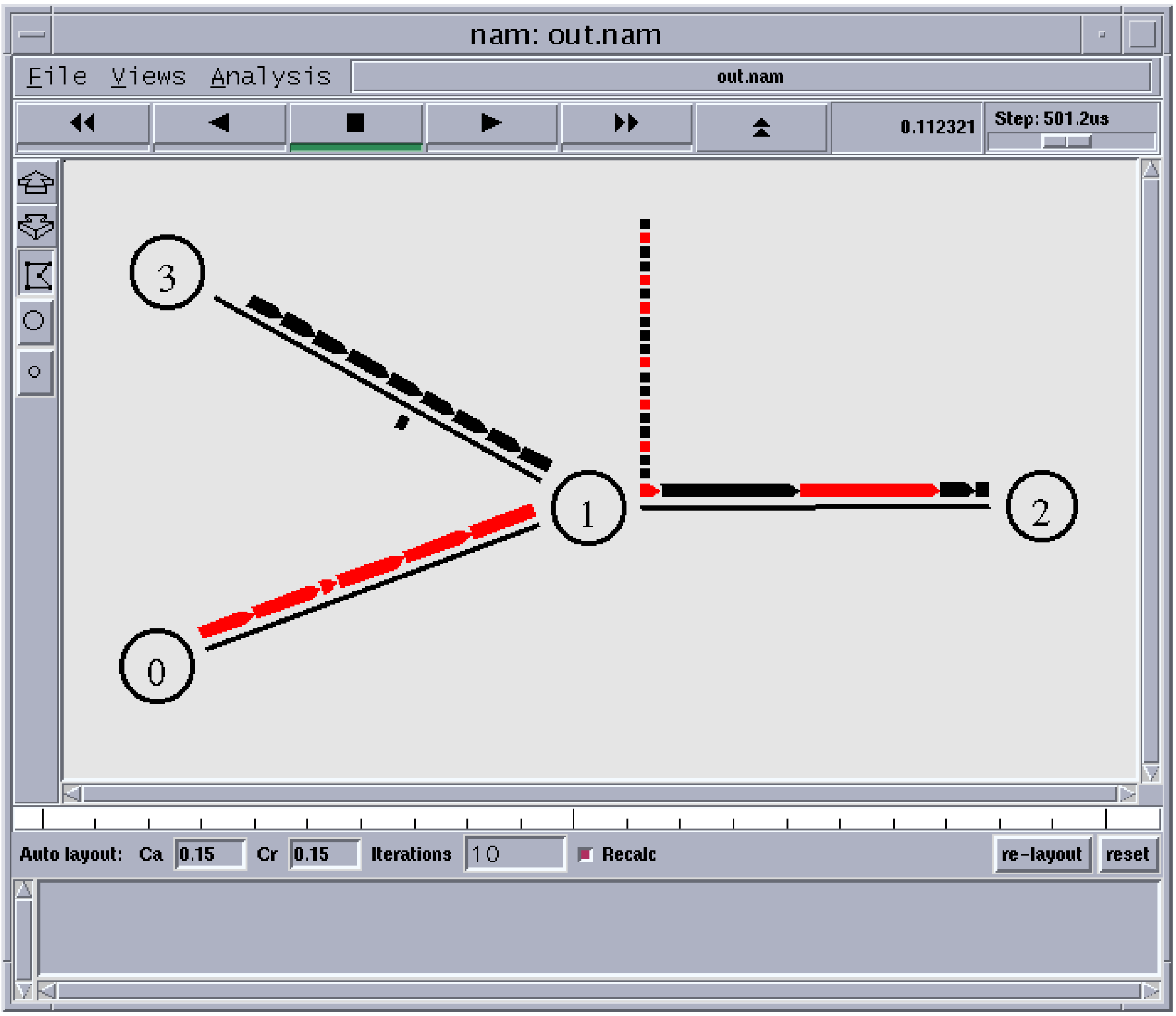
Sample-path behavior of queue length



Sample-path behavior of ER controller



Simulation (ns) : packet level



Link 1-2 has C=0.5 Mb/s

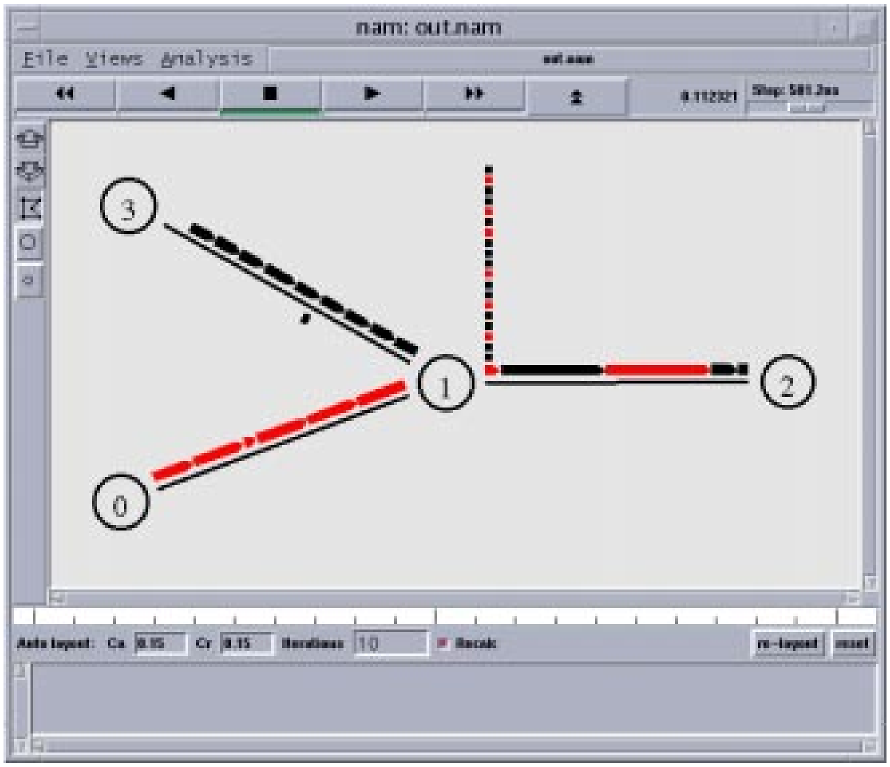
$d_{1,1} = 2ms \quad d_{2,1} = 10ms \quad d_2 = 4ms$

Source 1 (0) starts sending packets at 1 MB/s

Source 2 (3) starts sending packets at 2 MB/s

ER is initialized at 0.5 MB/s

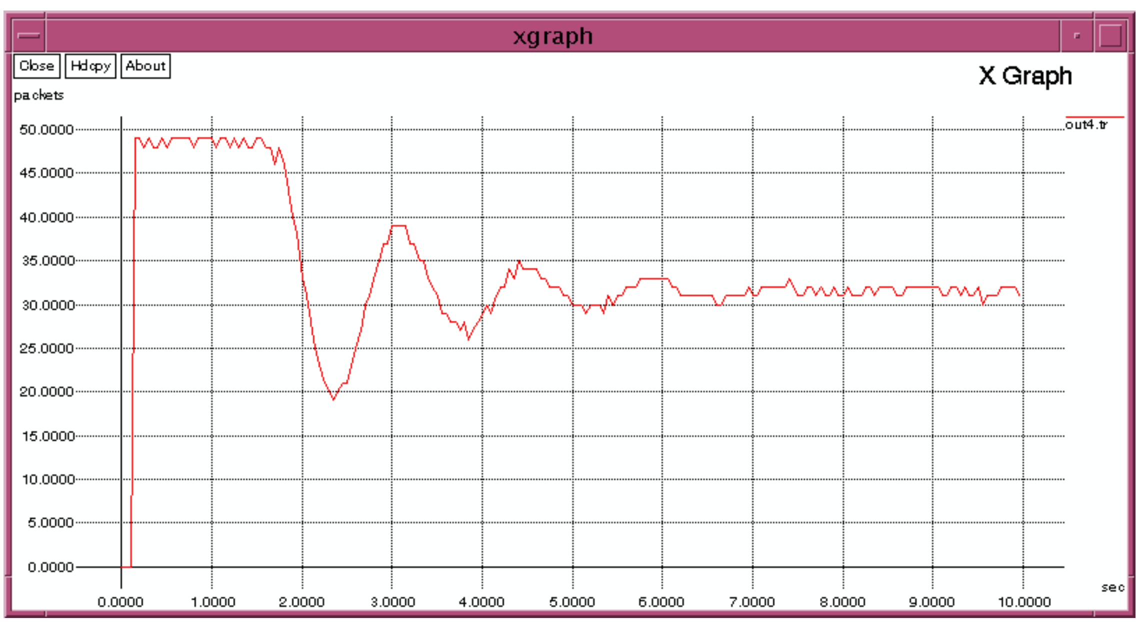
Every 20th packet is RM



Q=50 packets, Q^{*}=30 packets, 1 packet = 10^3 bytes

$(\alpha, \beta) = (0.05, 0.02)$

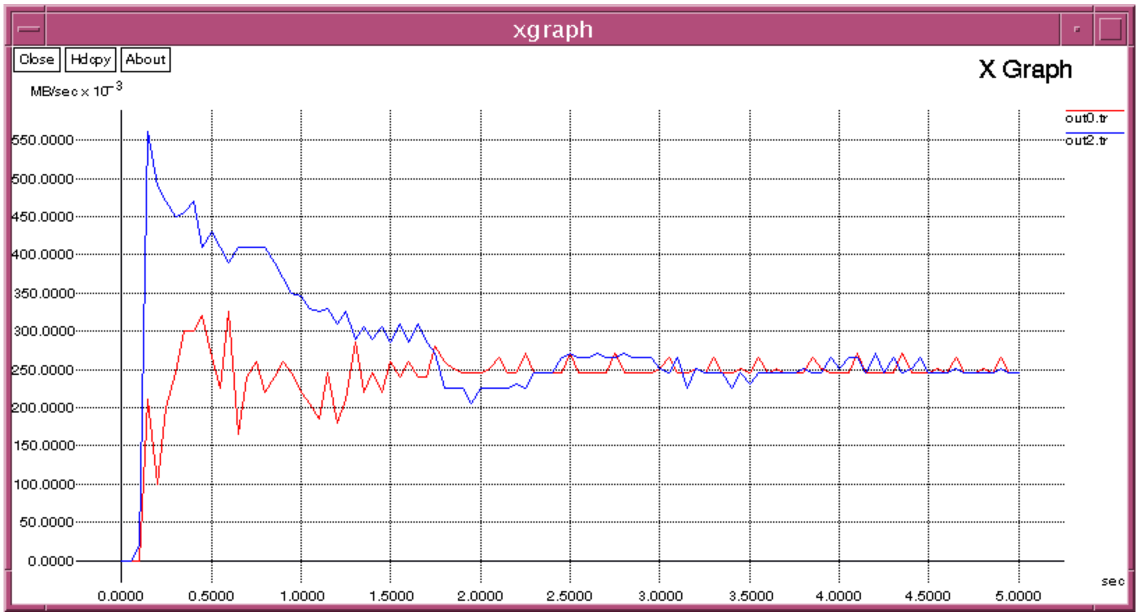
Queue Length



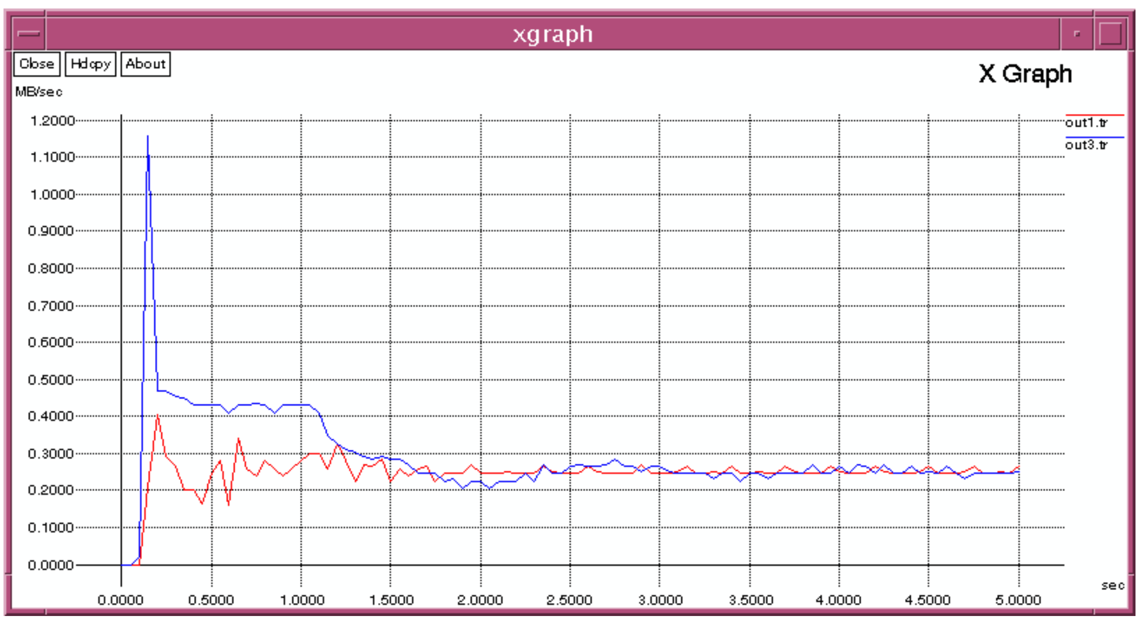
Flow rates of the two sources

Blue: From source **Red: To destination**

Source 1 $(d_{1,1} = 2ms)$



Source 2 $(d_{2,1} = 10ms)$



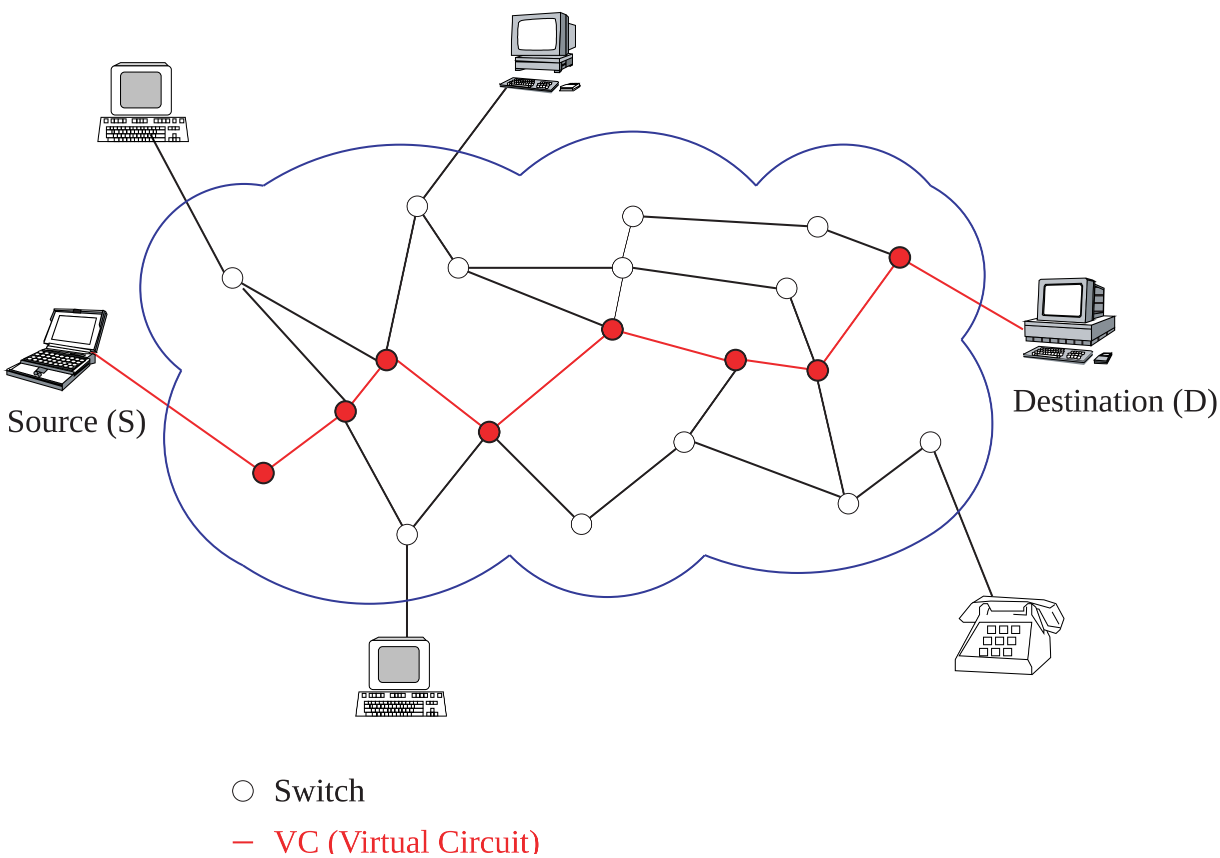
$r_1(\infty) = r_2(\infty) = 0.25$ $\quad$ $\mathrm{ER}(\infty) = 0.25$

6. Network Level Implementation of the ER Algorithm

Is $\lim_{n\to\infty} \mathcal{B}_l(n) = \mathcal{B}_l(\infty) \ \forall\, l \in \mathcal{L}$ **valid ?**

Simplifying assumptions (for clarity)

1. **No delay**
2. $\mathrm{MR}_s = 0 \ \forall\, s \in \mathcal{S}$
3. **Ignore saturation nonlinearities**
4. M_l **and network topology do not change with time**



$\Rightarrow$

$$q_l(n+1) = q_l(n) + \sum_{s \in \mathcal{S}_l} r_s(n) - C_l$$

$$\mathrm{ER}_l(n+1) = \mathrm{ER}_l(n) - \beta_l \left(q_l(n) - Q_l^*\right) - \alpha_l \left(\sum_{s \in \mathcal{S}_l} r_s(n) - C_l \right)$$

$$r_s(n) = \min_{l \in \mathcal{L}_s} \mathrm{ER}_l(n)$$

$\Rightarrow$

$$F_l(n) = \sum_{s \in \mathcal{B}_l(n)} r_s(n) + \sum_{s \in \mathcal{B}_l^c(n)} r_s(n) = M_{l,l}\, \mathrm{ER}_l(n) + \sum_{s \in \mathcal{B}_l^c(n)} \min_{l \in \mathcal{L}_s} \mathrm{ER}_l(n)$$

Overall system depends on the ordering of the ER's (L ! of them)

Each one corresponds to a different linear system

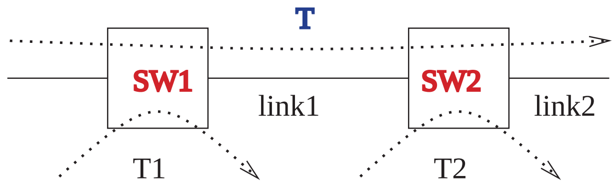
$\Rightarrow$ **A HYBRID SYSTEM**

Local Stability:

If every connection has at most 2 hops, the simplified ER algorithm is l.a.s. if

$0 < \alpha_l < 2/M_l, \quad 0 < \beta < \alpha_l\,.$

Global Analysis: Two-link case



$$M_{11}(n) = \begin{cases} M_1 - M_{1,2} & \mathrm{ER}_1(n) \geq \mathrm{ER}_2(n) \\ M_1 & \mathrm{ER}_1(n) \leq \mathrm{ER}_2(n) \end{cases}$$

$$M_{22}(n) = \begin{cases} M_2 & \mathrm{ER}_1(n) \geq \mathrm{ER}_2(n) \\ M_2 - M_{1,2} & \mathrm{ER}_1(n) \leq \mathrm{ER}_2(n) \end{cases}$$

$$\Rightarrow \quad \xi(n+1) = \begin{cases} \Pi_1 \xi(n) + \pi & \mathrm{ER}_1(n) \geq \mathrm{ER}_2(n) \\ \Pi_2 \xi(n) + \pi & \mathrm{ER}_1(n) \leq \mathrm{ER}_2(n) \end{cases}$$

$$\xi(n) = \left[q_1(n) \;\; q_2(n) \;\; \mathrm{ER}_1(n) \;\; \mathrm{ER}_2(n) \right]^T,$$
$$\pi = \left[-C_1 \;\; -C_2 \;\; \alpha_1 C_1 + \beta_1 Q_1^* \;\; \alpha_2 C_2 + \beta_2 Q_2^* \right]^T,$$

$$\Pi_1 = \begin{pmatrix} 1 & 0 & M_1 - M_{1,2} & M_{1,2} \\ 0 & 1 & 0 & M_2 \\ -\beta_1 & 0 & 1 - \alpha_1 (M_1 - M_{1,2}) & -\alpha_1 M_{1,2} \\ 0 & -\beta_2 & 0 & 1 - \alpha_2 M_2 \end{pmatrix}$$

$$\Pi_2 = \begin{pmatrix} 1 & 0 & M_1 & 0 \\ 0 & 1 & M_{1,2} & M_2 - M_{1,2} \\ -\beta_1 & 0 & 1 - \alpha_1 M_1 & 0 \\ 0 & -\beta_2 & -\alpha_2 M_{1,2} & 1 - \alpha_2 (M_2 - M_{1,2}) \end{pmatrix}$$

C_l: **available bandwidth at switch** l

(α_l, β_l): **controller parameters**

Decentralized implementation: switch l **has access to only** C_l **and** M_l**, as** $M_{1,2}$ **may not be known.**

Assume $\dfrac{C_1}{M_1} \geq \dfrac{C_2}{M_2}$, **Let** $g_1 := 1 - (M_{1,2}/M_1)$

$\Rightarrow$ **unique max-min fair equilibrium:**

$$q_{1e} = Q_1^*\,, \quad q_{2e} = Q_2^*,$$

$$\mathrm{ER}_{1e} = \frac{C_1 - C_2(M_{1,2}/M_2)}{M_1 - M_{1,2}}\,, \quad \mathrm{ER}_{2e} = \frac{C_2}{M_2}$$

$\Rightarrow$

$$x(n+1) = \begin{cases} \Pi_1 x(n) & x_4(n) - x_3(n) \leq \dfrac{\frac{C_1}{M_1} - \frac{C_2}{M_2}}{g_1} \\ \Pi_2 x(n) + b & x_4(n) - x_3(n) \geq \dfrac{\frac{C_1}{M_1} - \frac{C_2}{M_2}}{g_1} \end{cases}$$

Let

$$y_1(n) := \beta_1 x_1(n) - \beta_2 x_2(n)\,, \quad y_2(n) := x_4(n) - x_3(n)$$

Then,

$$y(n+1) = A_1 y(n) - B\Phi\big(\tilde{C}y(n) - \frac{a}{g_1}\big)$$

where

$$\Phi(z) = \begin{cases} 0 & z \leq 0 \\ z & z \geq 0 \end{cases}, \qquad A_1 = \begin{pmatrix} 1 & -\beta g_1 \\ 1 & 1 - \alpha g_1 \end{pmatrix}$$

Stability can be studied within the framework of absolute stability of sampled-data systems

Discrete-time Popov criterion

Lyapunov function:

$$V(y) = y^T P y + 2\delta \int_0^{\tilde{C}y - \frac{a}{g_1}} \Phi(s) ds$$
$$= y^T P y + \delta \Phi^2 (\tilde{C}y - \frac{a}{g_1})$$

Let Q and γ be defined through:

$$A_1^T \left(P + |\delta| A_1^T \tilde{C}^T \tilde{C}\right) A_1 - P = -QQ^T$$
$$B^T \left(P + |\delta| \tilde{C}^T \tilde{C}\right) A_1 = \gamma Q^T$$
$$-B^T \left(P + |\delta| \tilde{C}^T \tilde{C}\right) B + \delta = \gamma^2$$

Then,

$$\Delta V(y) \leq -\left[Q^T y + \gamma \Phi \left(\tilde{C}y - \frac{a}{g_1}\right)\right]^2$$

$\exists\ (\alpha, \beta)$ **such that** $\exists\ (\delta, \gamma, Q)$ **with** $P > 0$

$\Rightarrow$ **globally stable**

7. Extension to a Marking Based Scheme

Update of marking probabilities

Mark packets with probability: $1 - e^{\lambda_l(n)}$

$$\lambda_l(n+1) = \max\{0, \min\{C_l,\ \lambda_l(n) - \alpha_l(F_\ell(n) - C_l) - \beta_l(q_\ell(n) - Q_l^*)\}\}$$

- **REM (Random Exponential Marking)**

Fraction of unmarked packets measured by source s:

$$f_s(n) = \exp\left(-\sum_{l \in \mathcal{L}_s} \lambda_l(n)\right)$$

$$\Rightarrow \quad r_s(n) = \mathrm{MR}_s + g\left(\sum_{l \in \mathcal{L}_s} \lambda_l(n)\right)$$

Aggregate flow on link ℓ:

$$F_l(n) = \sum_{s \in \mathcal{S}_l} g\left(\sum_{k \in \mathcal{L}_s} \lambda_k(n)\right) + G_l$$

$\Longrightarrow$ **Update algorithm:**

$$q_l(n+1) = \max\left\{0, \min\left\{Q_l, q_l(n) + g\left(\sum_{k\in\mathcal{L}_s} \lambda_k(n)\right) - (C_l - G_l)\right\}\right\}$$

$$\lambda_l(n+1) = \max\{0, \min\{C_l, \lambda_l(n) - \beta_\ell(q_l(n) - Q_l^*) - \alpha_l\left(\sum_{s\in\mathcal{S}_l} g\left(\sum_{k\in\mathcal{L}_s} \lambda_k(n)\right) - (C_l - G_l)\right)\}\}$$

Use $g(z) = z^{-1}, z > 0$ $\Rightarrow$

$\Rightarrow$ **Leads to proportionally fair rates**

$$r_s(\infty) = \mathrm{MR}_s + \frac{1}{\sum_{k\in\mathcal{L}_s} \lambda_k(\infty)}$$

Relationship with utility maximization

$$\max_{\{x_s \geq \mathrm{MR}_s\}} \sum_{s \in \mathcal{S}} U_s(x_s)$$

subject to $$\sum_{s \in \mathcal{S}_l} x_s \leq C_l\,, \;\; \forall\, l \in \mathcal{L}$$

$U_s(x_s)$ **strictly concave increasing, and** $\mathcal{C}^2$

Gradient projection applied to the dual problem:

$$\lambda_l(n+1) = \max\{0, \lambda_l(n) + \gamma_l(F_l(n) - C_l)\}$$

$$x_s(n) = g_s\Big(\sum_{l \in \mathcal{S}_l} \lambda_l(n)\Big)\,, \;\; \forall\, s \in \mathcal{S}$$

$$g_s(z) = \max\{\mathrm{MR}_s, U_s'^{-1}(z)\}$$

If γ_l is sufficiently small, then $x_s(n) \to$ optimal sol., but leads to queue build-up.

A better one is the REM algorithm

$$\lambda_l(n+1) = \max\{0, \min\{C_l, \lambda_l(n) - \alpha_l(F_\ell(n) - C_l) \\ - \beta_l(q_\ell(n) - Q_l^*)\}\}$$

which avoids queue build-up.

8. Recap

- Saturation-type nonlinearities
- Hybrid systems
- Delay and uncertainty
- Decentralized
- Distributed
- Asymptotics
- Economics
- Games
- • • • •

$\Longleftarrow$ $\Longrightarrow$ **FEEDBACK**

A FERTILE GROUND FOR CONTROL !