Control over Unreliable Communication Links

(based on joint work with O.C. Imer, S.Yüksel)

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Outline

- Networks & Control
- Issues in Networked Control Systems

  -- What/when/how to transmit & control
- Control with Unreliable Channels
- Control with Power-Limited Communication

  -- When to measure & when to control
- Conclusions

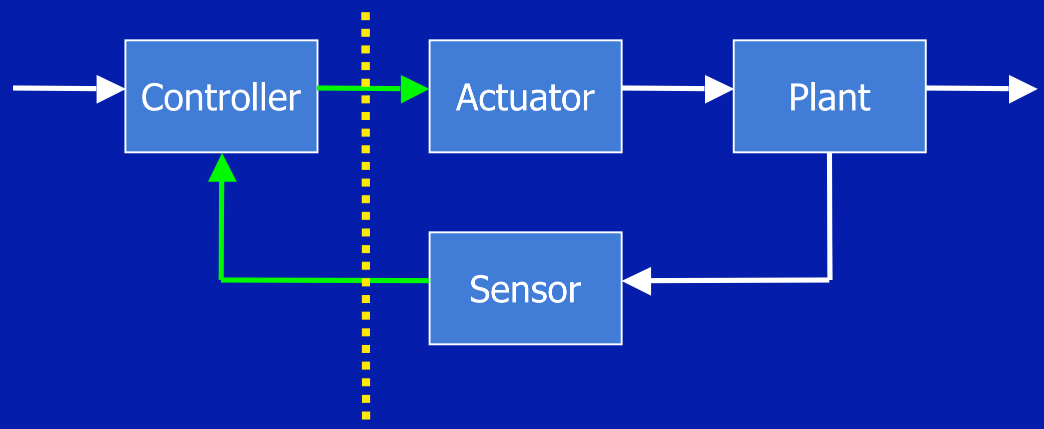
Networks & Control

- A surge of research interest in using networks for communication in control/monitoring systems
- Advantages: flexibility, reduced wiring, lower installation costs, agility in diagnosis and maintenance, remote operations, ...
- Challenges: synchronization, timing problems, reliability, *communication network and channel constraints*, ...

Networks & Control

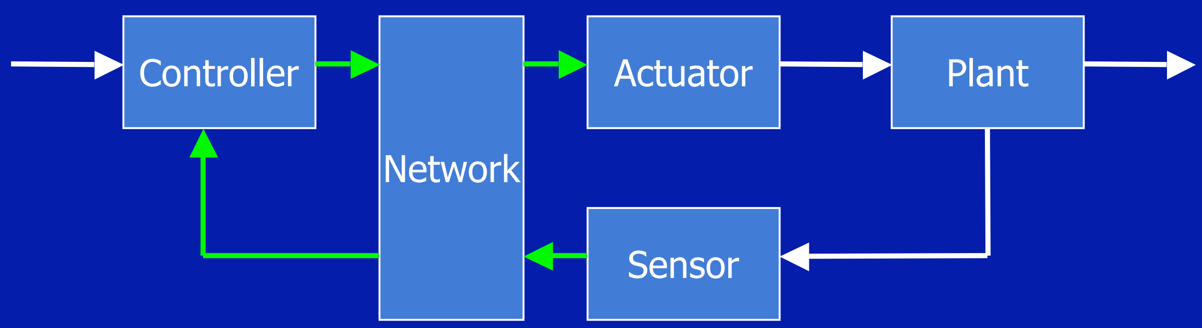
- Traditional Control Systems have dedicated data paths for communication

Controller-Plant Communication

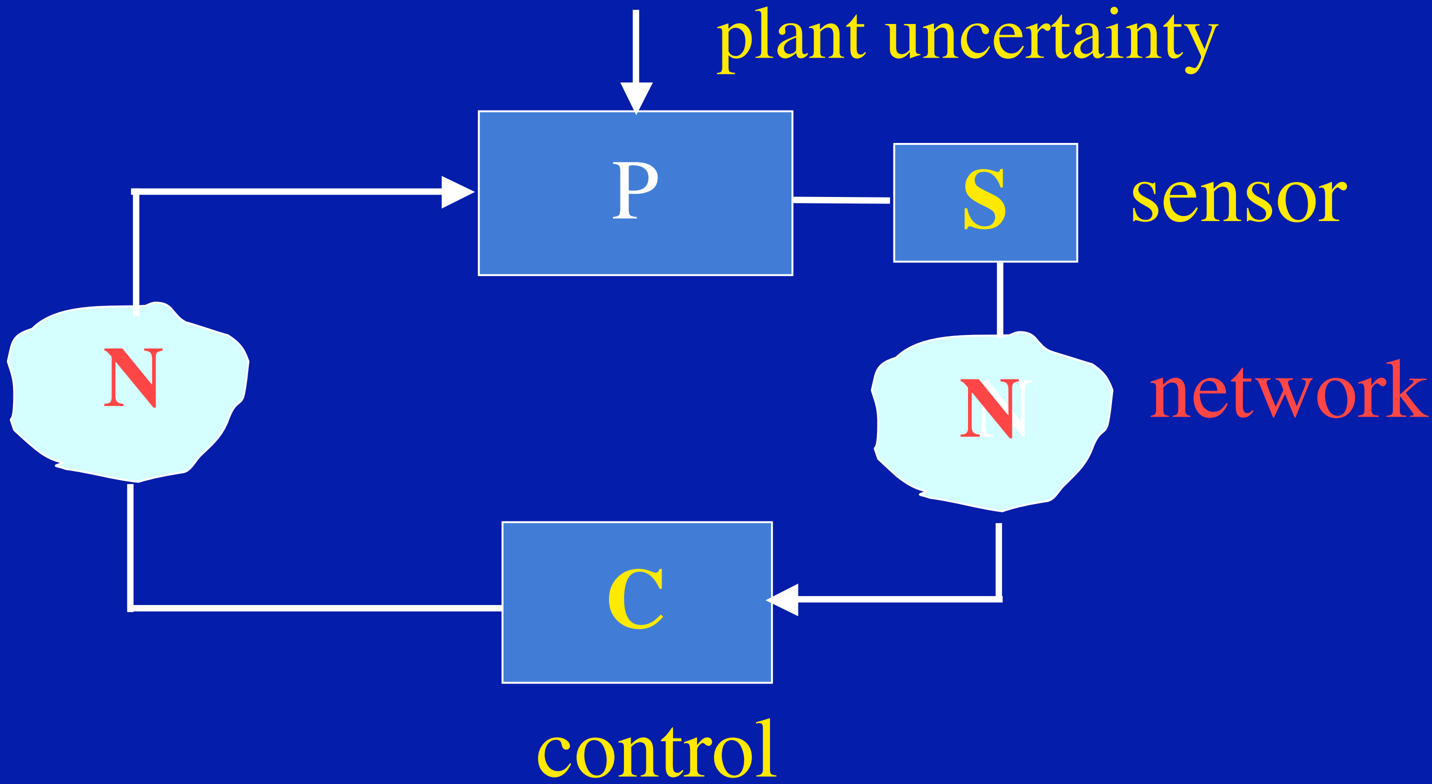


Networks & Control

- In Networked Control Systems (NCS) controller-plant communication takes place over a network



Remote Control Paradigm



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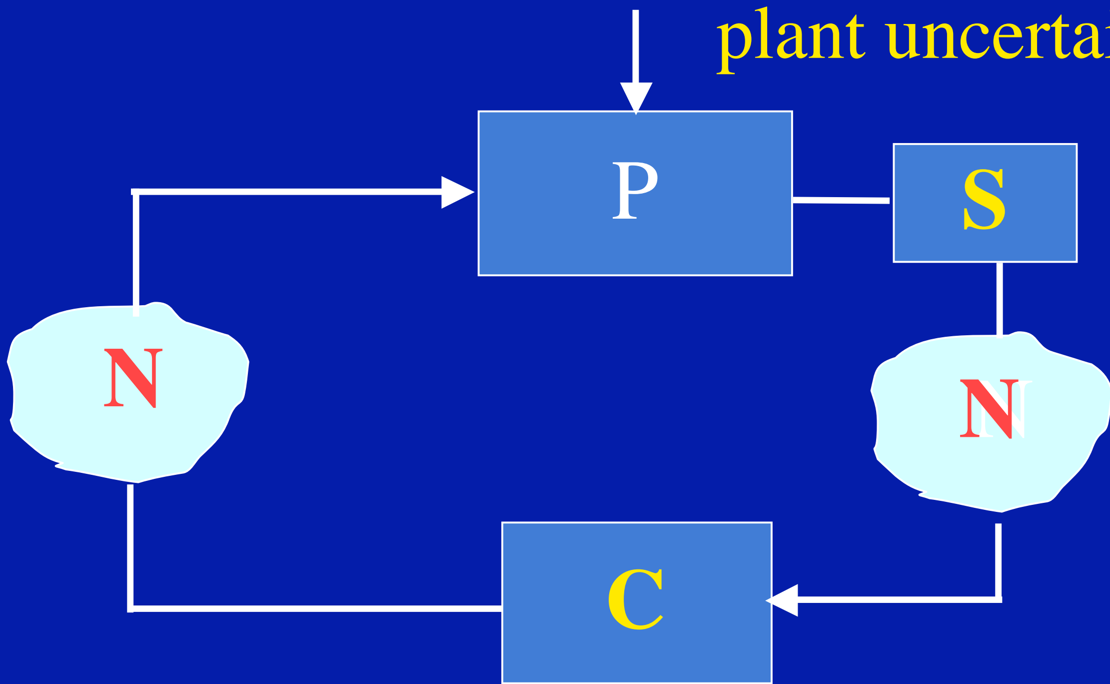
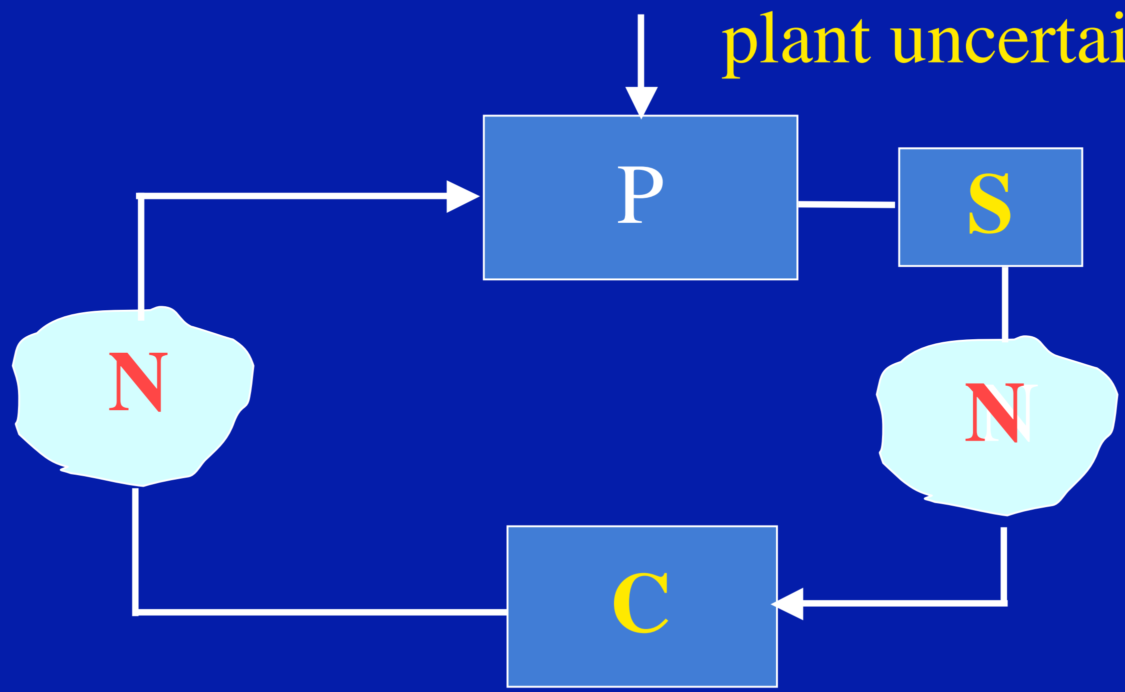


network
BW limitations
Usage limitations
Delay
Quantization (Q)
Link failures

PI → optimize

Remote Control Paradigm



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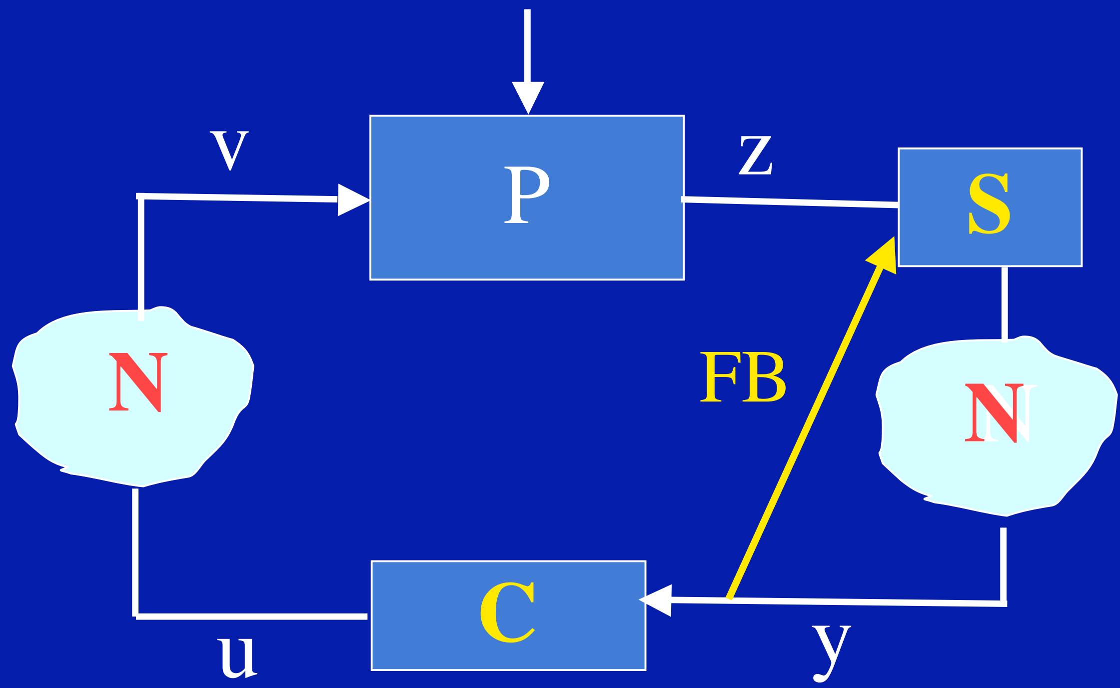

sensor

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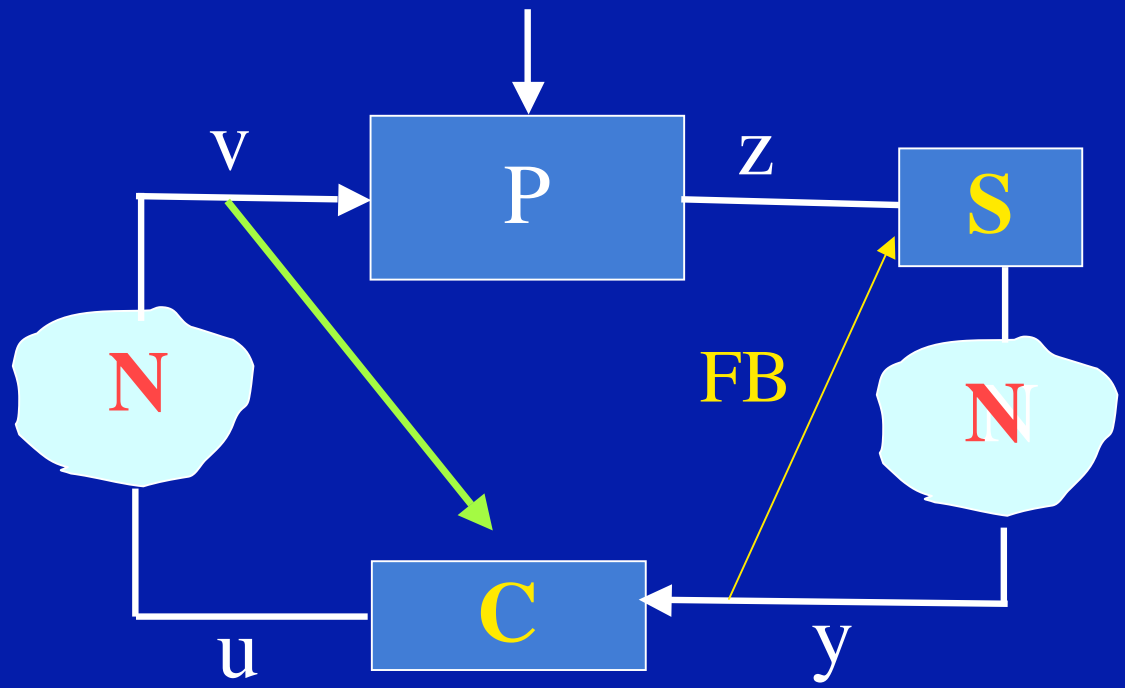

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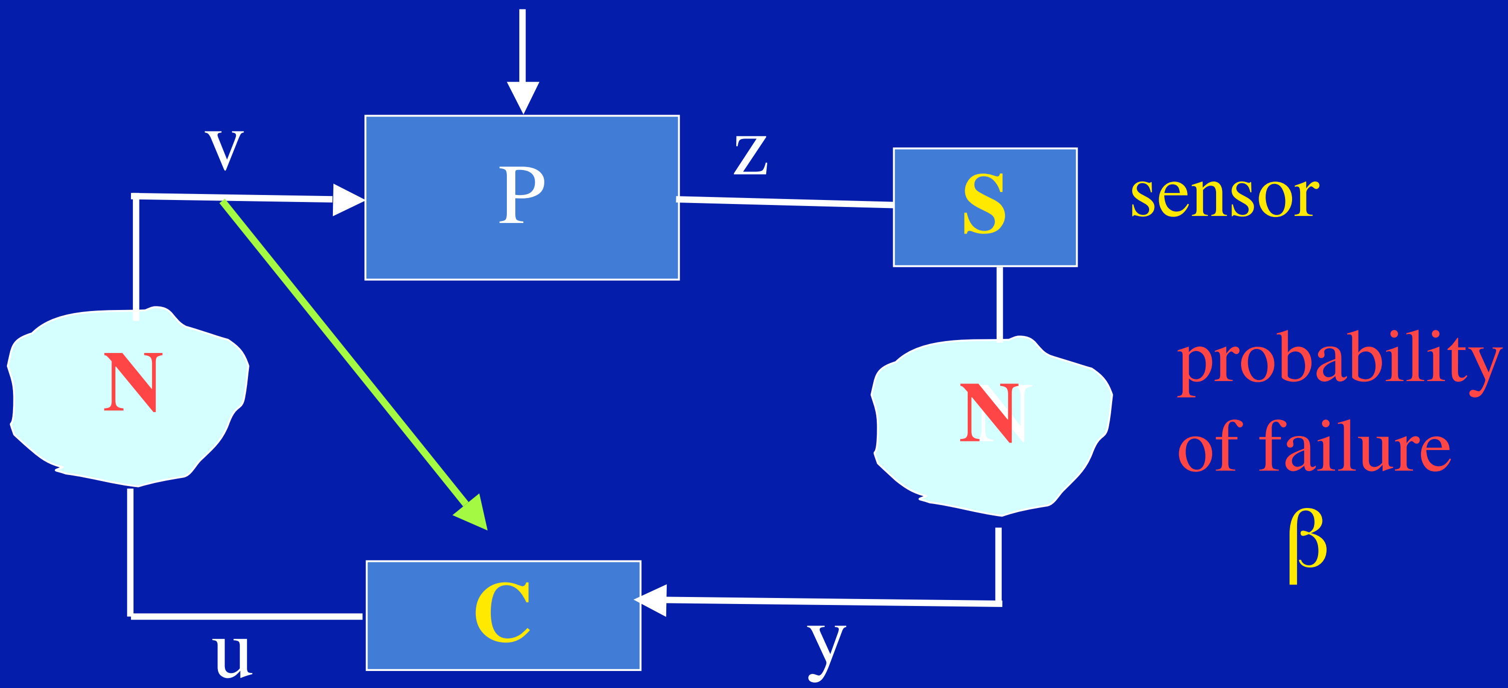
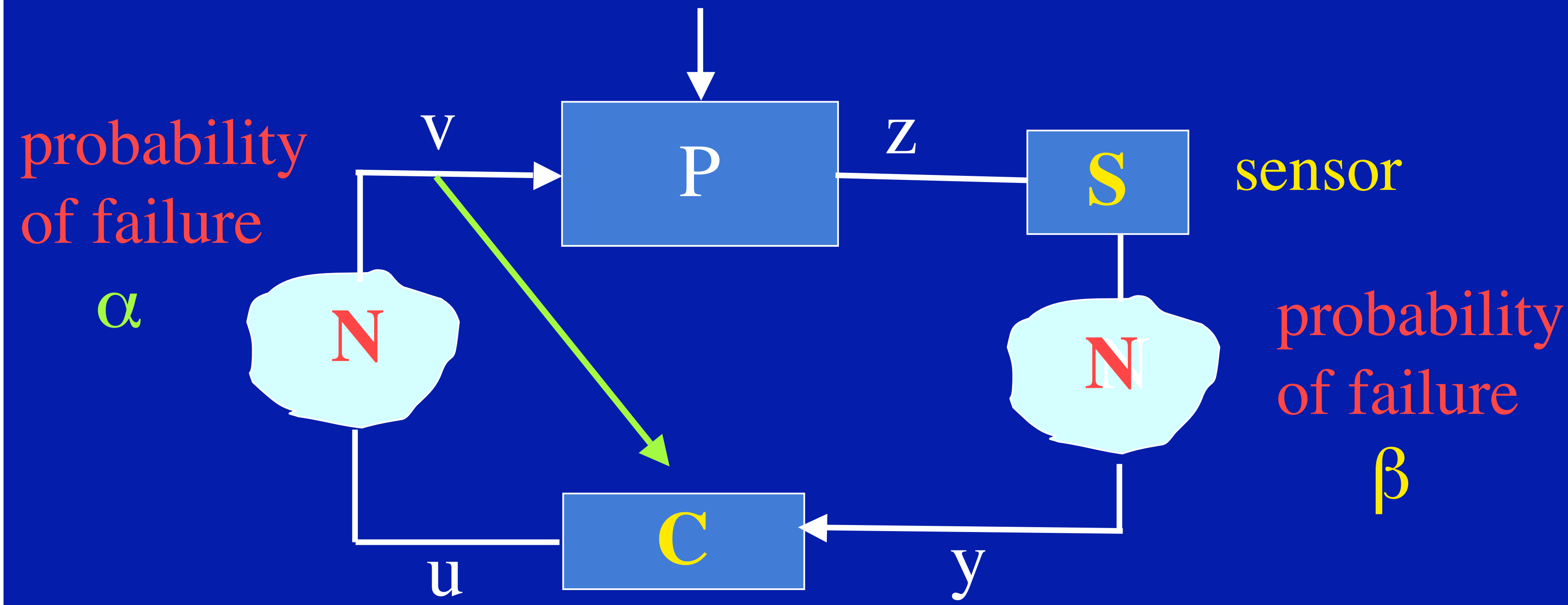


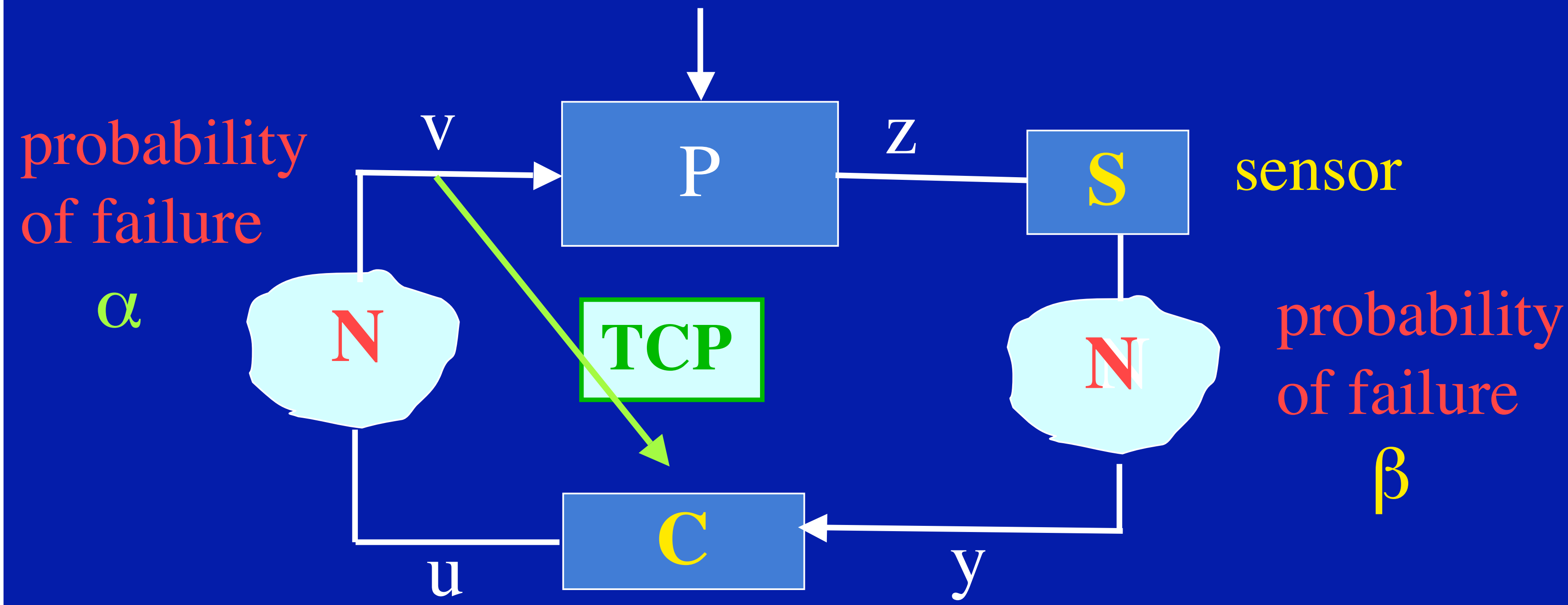
PI → optimize

Link Failures



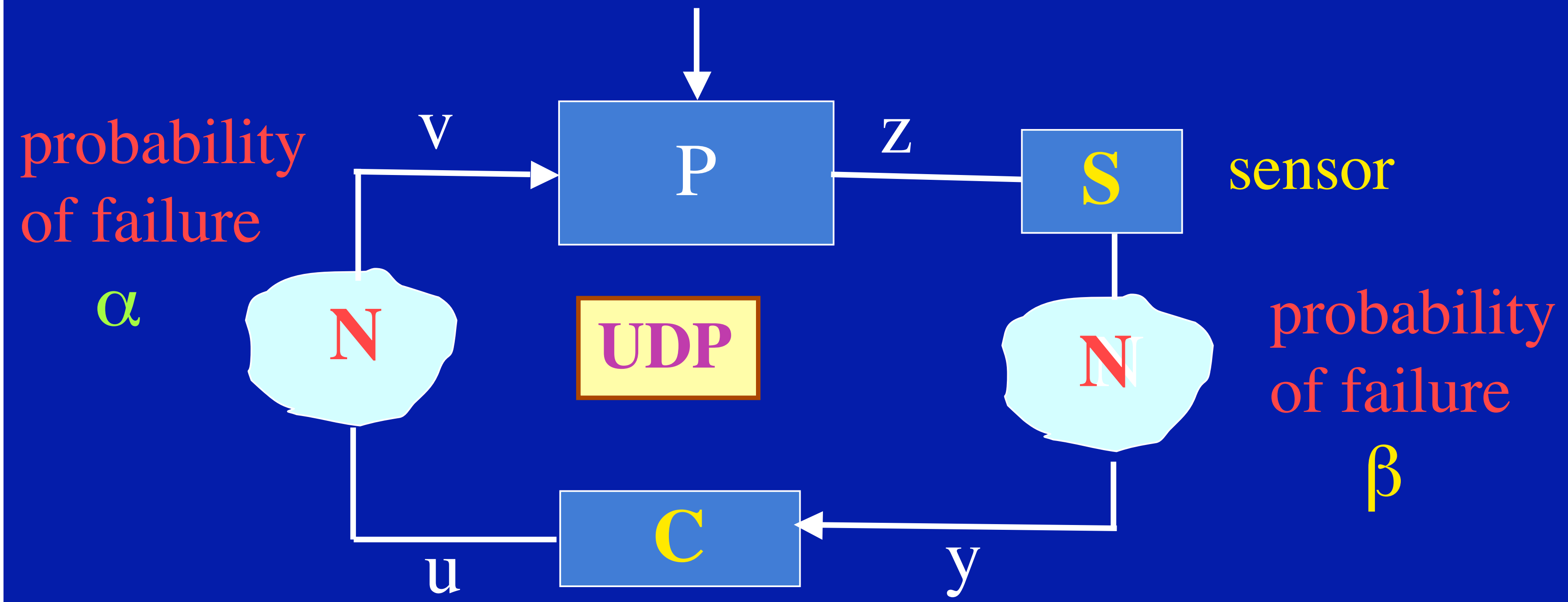
PI → optimize

Scenario I



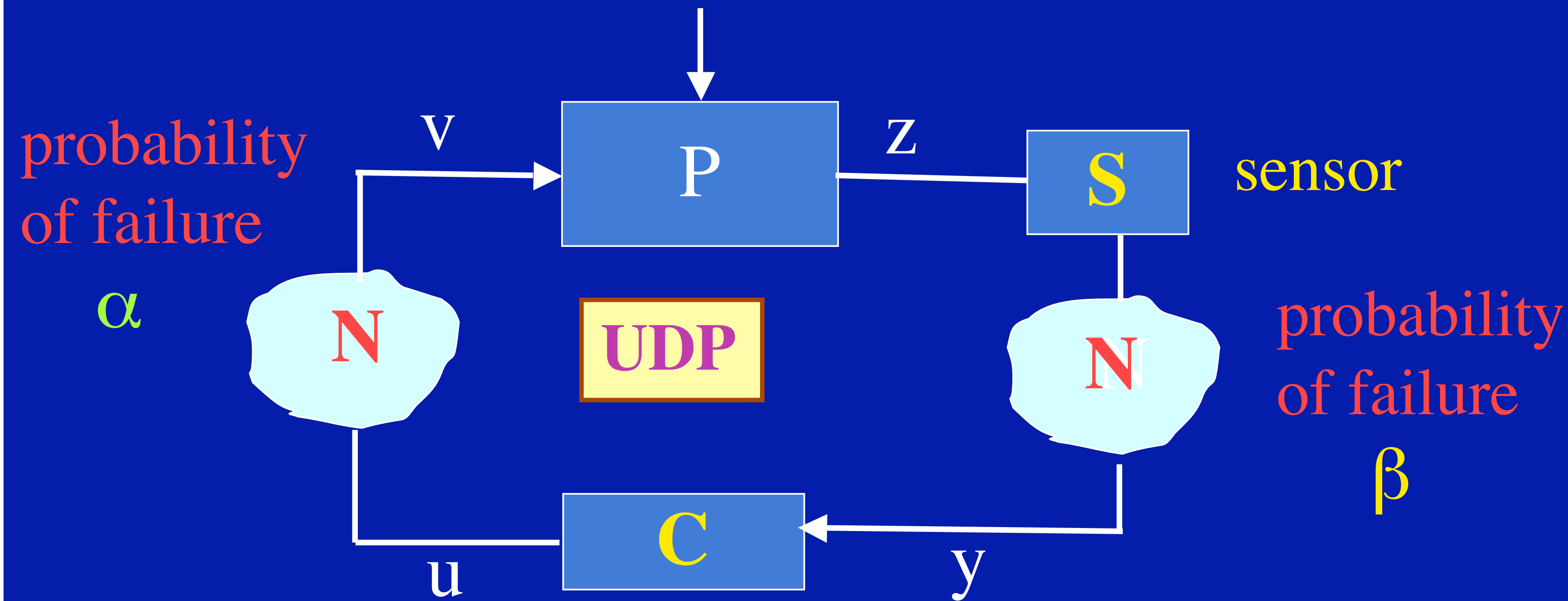
Transmission Control Protocol of Internet
(with acknowledgements)

Scenario II



User Datagram Protocol -- best effort network
(no acknowledgements)

Scenario II



$$I_k^{UDP} = \{y_0^k,\ u_0^{k-1},\ \beta_0^k\}$$
$$u_k = \mu_k\,(I_k^{UDP})$$

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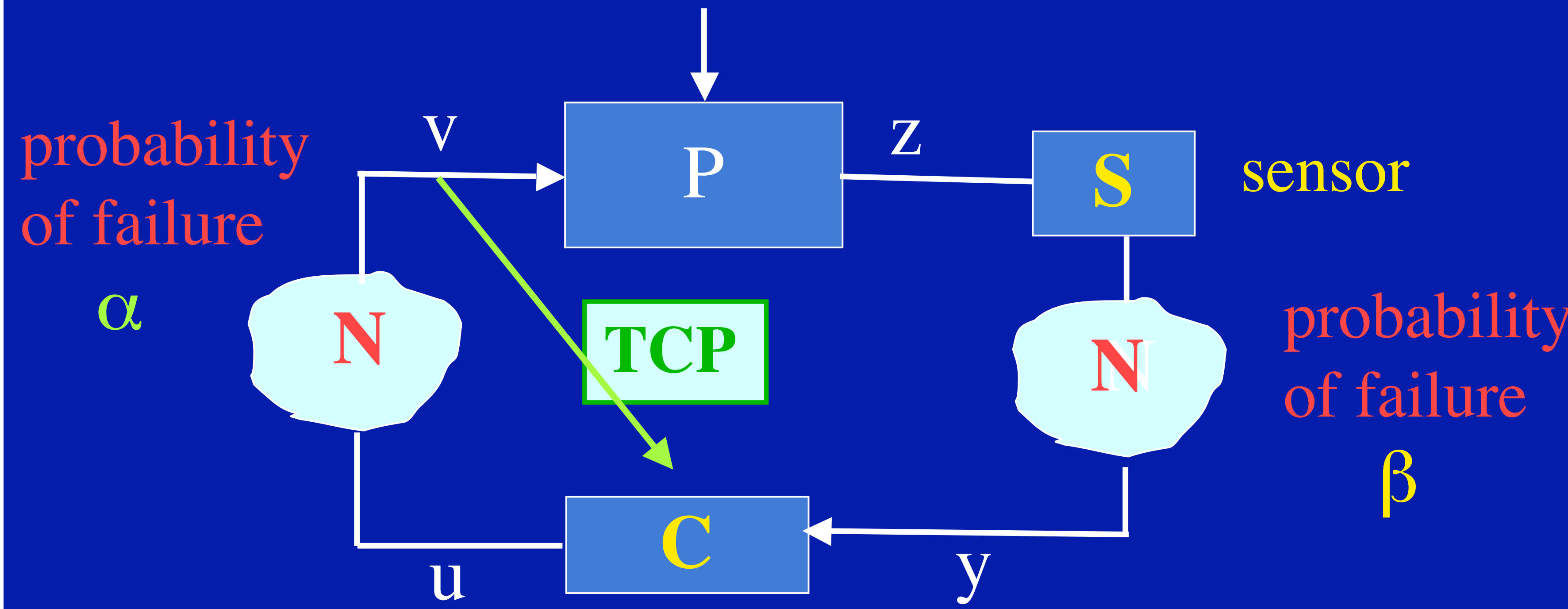

$$I_k^{TCP} = \{I_k^{UDP},\ \alpha_0^{k-1}\}$$
$$u_k = \mu_k\,(I_k^{TCP})$$

Mathematical Formulation

$x_{k+1} = f(x_k, v_k, w_k), \quad k = 0,1,\ldots$

$v_k = \alpha_k u_k$ or $v_k = v_{k-1}$ if $\alpha_k = 0$

$y_k = \beta_k z_k$ $\quad z_k = h(x_k, w_k)$

$\{\alpha_k\}, \{\beta_k\}$ independent *i.i.d.* Bernoulli

$\mathrm{Prob}(\alpha_k = 0) = \alpha$, $\quad \mathrm{Prob}(\beta_k = 0) = \beta$

$\{w_k\}$ *i.i.d.* plant / channel noise

Mathematical Formulation

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Control : $u_k = \mu_k (I_k^{TCP})$

or $u_k = \mu_k (I_k^{UDP})$

Mathematical Formulation

$x_{k+1} = f(x_k, v_k, w_k), \quad k = 0,1,\ldots$

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PI : $E_{\mu} \{q(x_N) + \sum_k g(x_k, v_k)\} =: J(\mu_0^N, N)$

or $\limsup_{N\to\infty} (1/N)\, J(\mu_0^N, N)$

LQG with erasure channels

$x_{k+1} = Ax_k + \alpha_k Bu_k + w_k$, $\quad k = 0,1,\ldots$

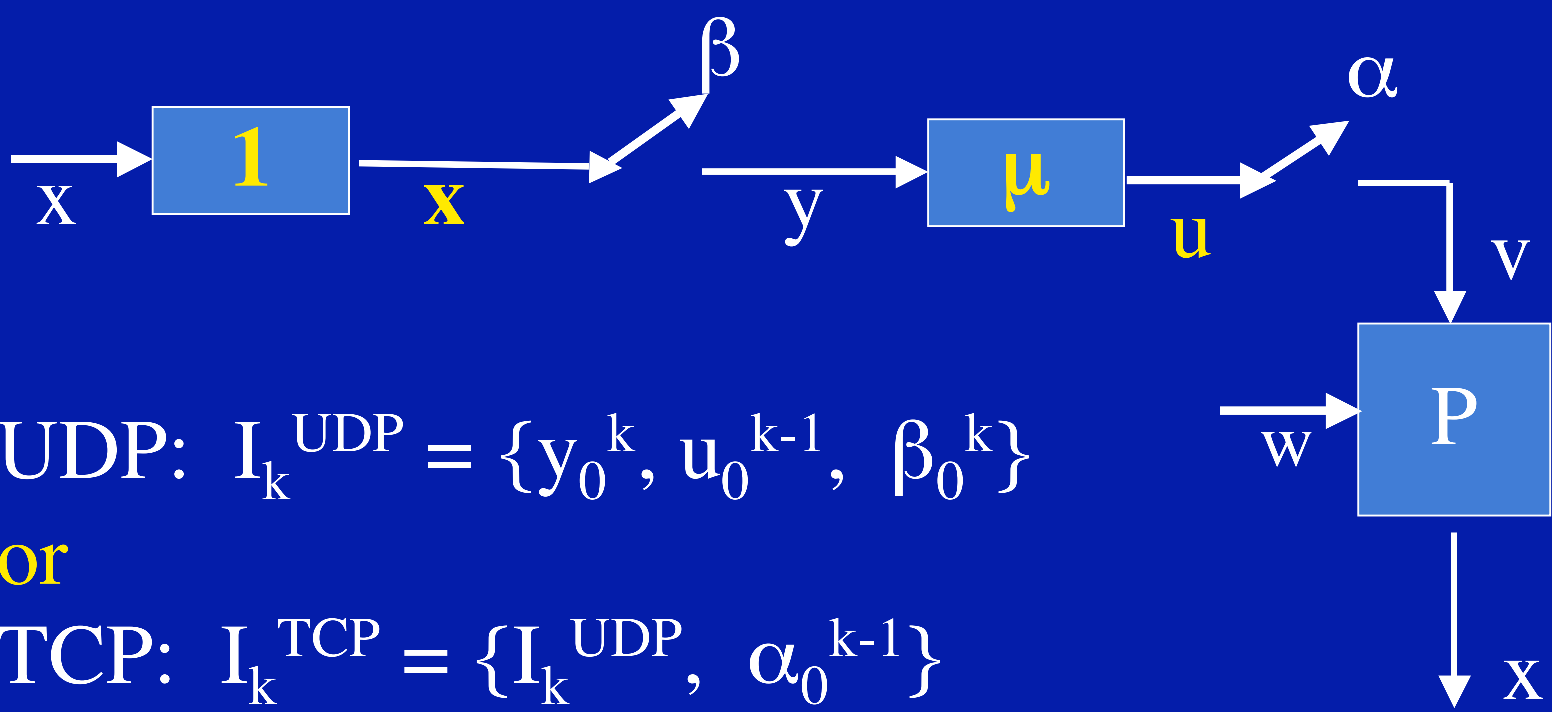
$y_k = \beta_k x_k \qquad z_k = x_k$

$\mathrm{Prob}(\alpha_k = 0) = \alpha, \qquad \mathrm{Prob}(\beta_k = 0) = \beta$

$J(\mu_0^N, N) = E_\mu \{|x_N|_F^2 + \sum_k |x_k|_Q^2 + \alpha_k |u_k|_R^2\}$

or $\quad \lim\sup_{N\to\infty} (1/N)\, J(\mu_0^N, N)$

LQG with erasure: Two Scenarios



UDP: $I_k^{UDP} = \{y_0^k, u_0^{k-1}, \beta_0^k\}$

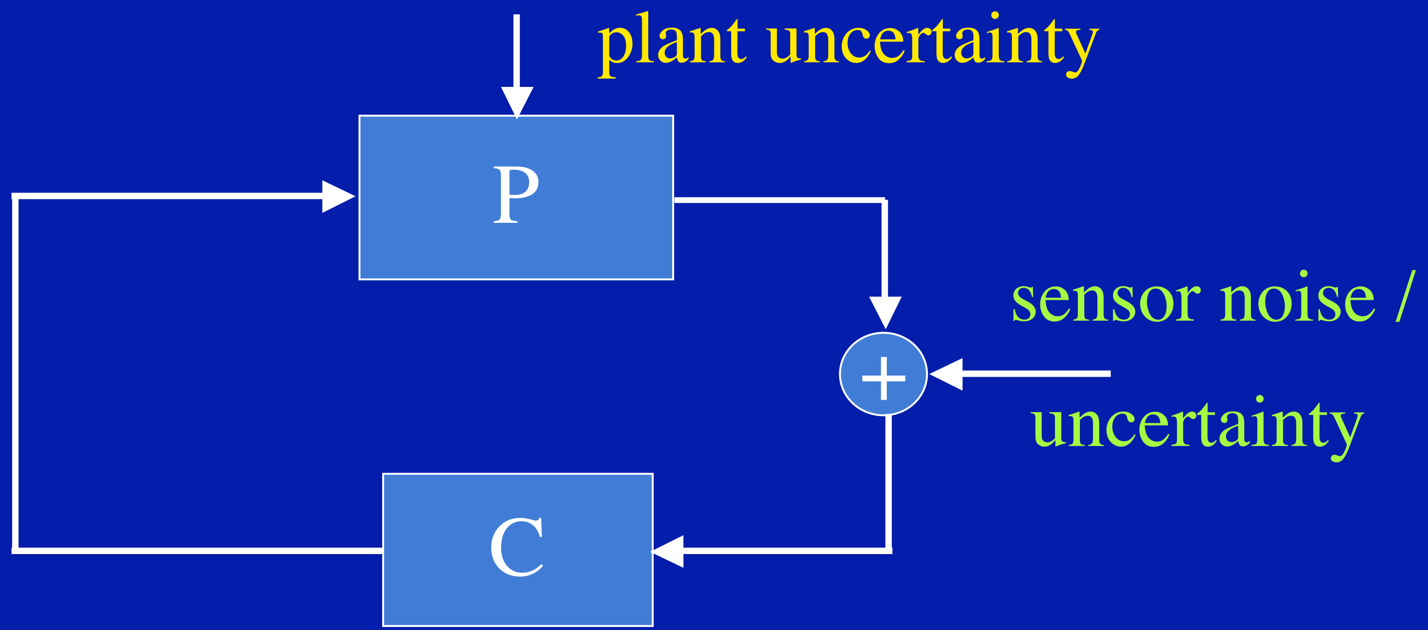
or

TCP: $I_k^{TCP} = \{I_k^{UDP}, \alpha_0^{k-1}\}$

$u_k = \mu_k (I_k)$

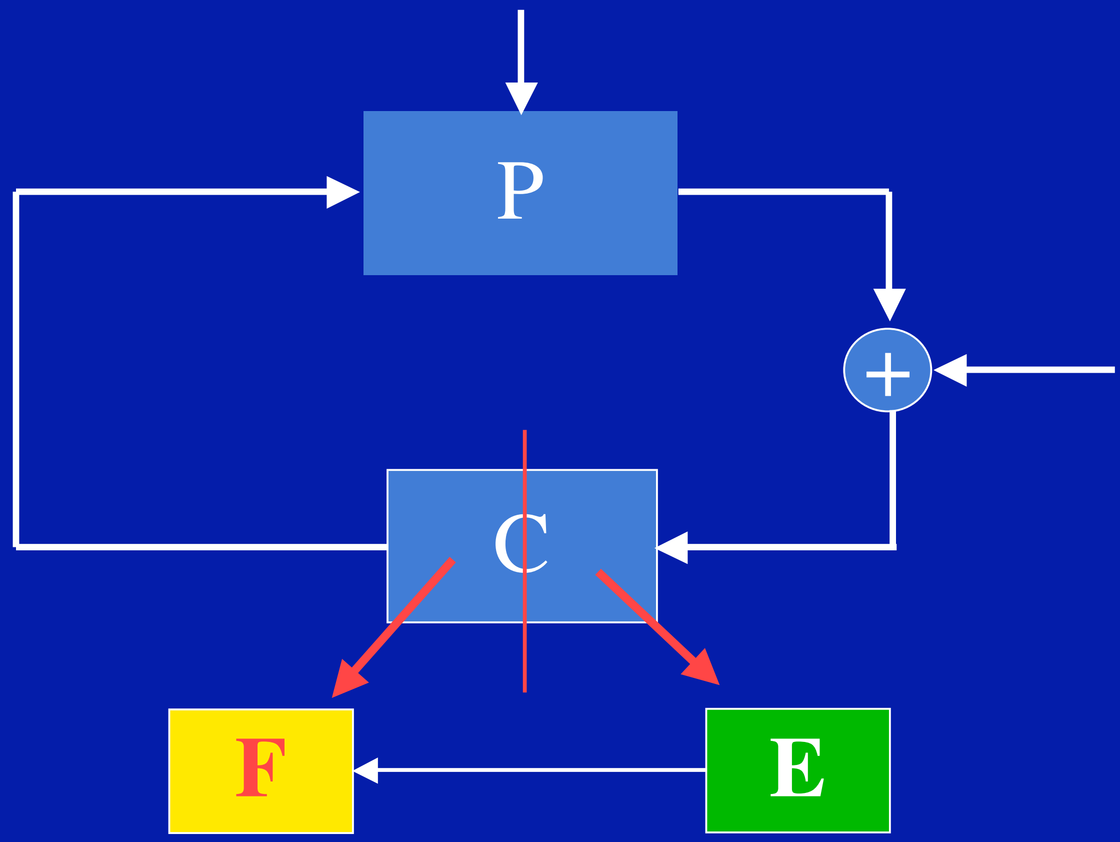
PI $= E_\mu \{|x_N|_F^2 + \sum |x_k|_Q^2 + \alpha_k |u_k|_R^2\}$

Digression: General Controller Design



dual role of control: action & probing
generally not aligned

Standard LQG: **Separation / Neutrality**



Back to LQG with Erasure With TCP: Finite Horizon

With TCP: Finite Horizon

Separation of estimation and control
no dual effect

With TCP: Finite Horizon

- Separation of estimation and control; no dual effect
- $u_k = G_k \, E[x_k \mid I_k]$
  $= - (R+B^T K_{k+1} B)^{-1} B^T K_{k+1} A \, E[x_k \mid I_k]$

$$E[x_k \mid I_k] = A \, E[x_{k-1} \mid I_{k-1}] + \alpha_{k-1} B u_{k-1} \quad \text{if } \beta_k = 0$$
$$= x_k \quad \text{if } \beta_k = 1$$

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If noisy measurements, replace with

KF with intermittent measurements

With TCP: Finite Horizon

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$$K_k = Ric_{TCP}(K_{k+1}) \qquad K_N = F$$

$$Ric_{TCP}(K) = A^T K A + Q - (1-\alpha)\, A^T K B (R+B^T K B)^{-1} B^T K A$$

With TCP: Infinite Horizon

$$u_k = G\, E[x_k \mid I_k]$$
$$= -\,(R+B^T K B)^{-1} B^T K A\, E[x_k \mid I_k]$$

$$K \leftarrow Ric_{TCP}(K) \;\Rightarrow\; K = Ric_{TCP}(K)$$

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If (A,Q) is observable, unique limit $K > 0$ iff

$M \leftarrow A^T M A - (1-\alpha)\, A^T M B (B^T M B)^{-1} B^T M A$

converges from $M_0 = I$ (based on Koning'82)

With TCP: Infinite Horizon

$u_k = G\, E[x_k \mid I_k]$

$= - (R+B^T K B)^{-1} B^T K A\, E[x_k \mid I_k]$

If B is square and invertible, condition reduces to

$|\lambda (A)| < 1 / \sqrt{\alpha}$

If (A,Q) is observable, unique limit $K > 0$ iff

$M \leftarrow A^T MA - (1- \alpha) A^T MB(B^T MB)^{-1} B^T MA$

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With TCP: Infinite Horizon

$u_k = G\, E[x_k \mid I_k]$

$\quad = -(R+B^T K B)^{-1} B^T K A\, E[x_k \mid I_k]$

Control is m.s. stabilizing if, in addition,

$|\lambda(A)| < 1/\sqrt{\beta}$

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No separation of estimation and control
Dual effect!

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But, estimator can still be designed separately:

$u_k = G_k(\alpha, \beta)\, E[x_k \mid I_k]$

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With UDP: Finite Horizon

No separation of estimation and control

Dual effect!

But, estimator can still be designed separately:

$u_k = G_k(\alpha, \beta)\ E[x_k \mid I_k]$

And G involves two recursions ($P_N = 0$, $K_N = F$):

$$P_k = \beta\ A^T P_{k+1} A - (1-\alpha)\ A^T K_{k+1} B G_k$$

$$K_k = A^T K_{k+1} A - P_{k+1} + \beta\ A^T P_{k+1} A + Q$$

$$G_k = -(R + B^T(K_{k+1} + \alpha\ \beta\ P_{k+1})B)^{-1} B^T K_{k+1} A$$

With UDP: Infinite Horizon

$$P \leftarrow \beta A^T P A - (1-\alpha) A^T K B G$$

$$K \leftarrow A^T K A - P + \beta A^T P A + Q$$

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Simpler test, equivalent condition if (A,Q) observable

(Q = R = 0, F = I in the above)

$$\Pi \leftarrow (1-\alpha) A^T \Lambda B (B^T(\Lambda + \alpha\beta\Pi)B)^{-1} B^T \Lambda A + \beta A^T \Pi A \qquad \Pi_0 = 0$$

$$\Lambda \leftarrow -(1-\alpha) A^T \Lambda B(B^T(\Lambda + \alpha\beta\Pi)B)^{-1} B^T \Lambda A + A^T \Lambda A \qquad \Lambda_0 = I$$

With UDP: Infinite Horizon

$$P \leftarrow \beta A^T P A - (1-\alpha) A^T K B G$$

$$K \leftarrow A^T K A - P + \beta A^T P A + Q$$

$$G = -(R + B^T(K + \alpha \beta P)B)^{-1} B^T K A$$

If Π and Λ converge, so do P and K, and G is the optimal steady-state gain

$$\Pi \leftarrow (1-\alpha) A^T \Lambda B (B^T(\Lambda + \alpha \beta \Pi)B)^{-1} B^T \Lambda A + \beta A^T \Pi A \qquad \Pi_0 = 0$$

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With UDP: Infinite Horizon

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$$K = A^T K A - P + \beta A^T P A + Q$$

$$G = -(R + B^T(K + \alpha\beta P)B)^{-1} B^T K A$$

Then, the steady-state optimal control is

$$u_k = G(\alpha, \beta) E[x_k \mid I_k]$$

$$\Pi \leftarrow (1-\alpha) A^T \Lambda B (B^T(\Lambda + \alpha\beta\Pi)B)^{-1} B^T \Lambda A + \beta A^T \Pi A \qquad \Pi_0 = 0$$

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Extensions

- Actuator applies last available control (instead of “zero”) when the control packet is lost.

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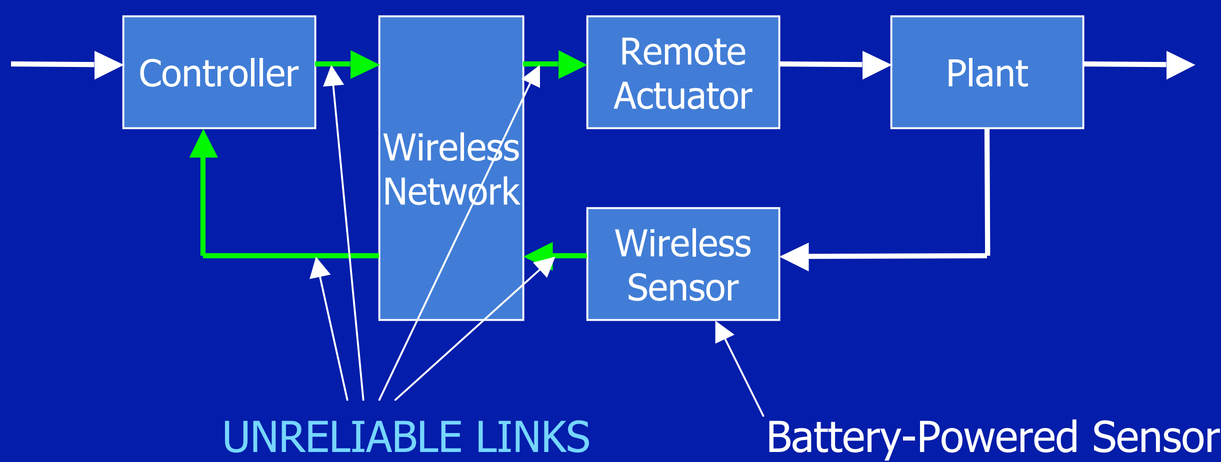
- Noisy measurements
- Correlated link failure processes
- With multiple channels, packets being lost with different probabilities on each sensor and actuator link
- Delays in the acknowledgements with TCP

Other Set-ups

Usage-Constrained Estimation and Control

Wireless Networks & Control

- Estimation & Control when controller-plant communication takes place over a Wireless Network



Wireless Networks & Control

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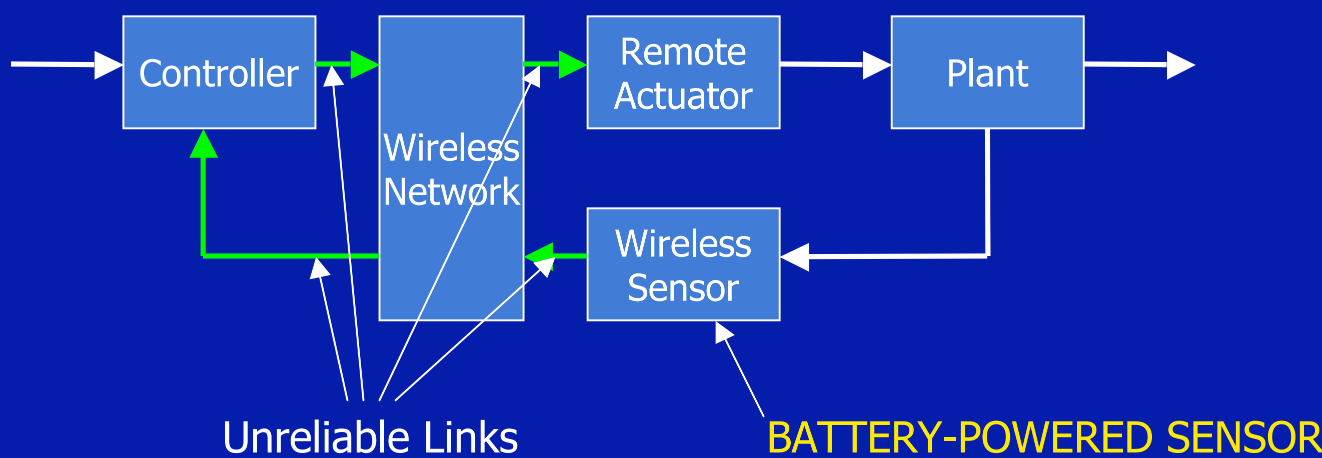

Limited Usage in Sensing & Control

Reasons

- To conserve battery power in wireless sensors
  (with RF channels a significant amount of power is consumed to transmit sensor measurements)
- Sharing of bandwidth among several users
  (time allocation limited transmission -- TDM systems)
- Power limitation on controller
- Limits on the frequency of interaction with the plant/system
- CANs -- actuator, controller, sensor connected over a serial bus -- access one at a time

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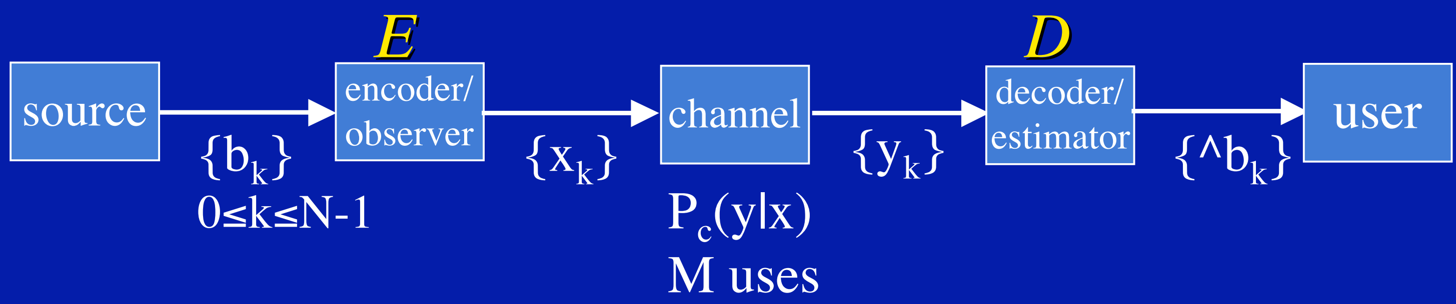
Sensing and Control are Expensive !

Three Classes of Problems

PROBLEM CLASS I

Optimal Estimation over a Limited-Use Channel



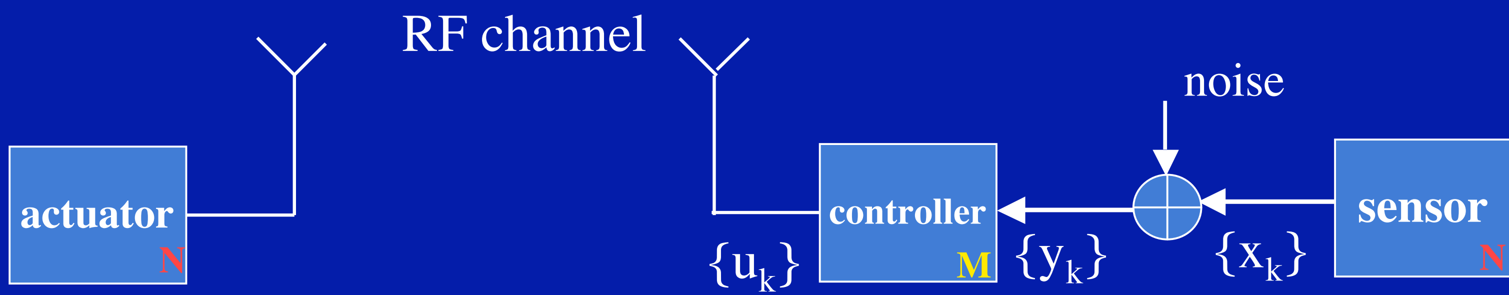
$x_k = \mathcal{E}(z_k)$ $\quad x \in \mathcal{X}$ $\quad y \in \mathcal{Y}$

$z_k = b_k + v_k$ $\quad M < N$

*Given a "source" and a "memoryless channel", for a given message length N, and number of channel uses M, what is the minimum attainable value of the average distortion $D_{(M,N)}$ and a corresponding **E & D** pair?*

PROBLEM CLASS II

Control over a Limited-Use Channel



$x_{k+1} = f(x_k, u_k, w_k)$ $x_k \in X_k$, $u_k \in U_k$, $y \in Y_k$

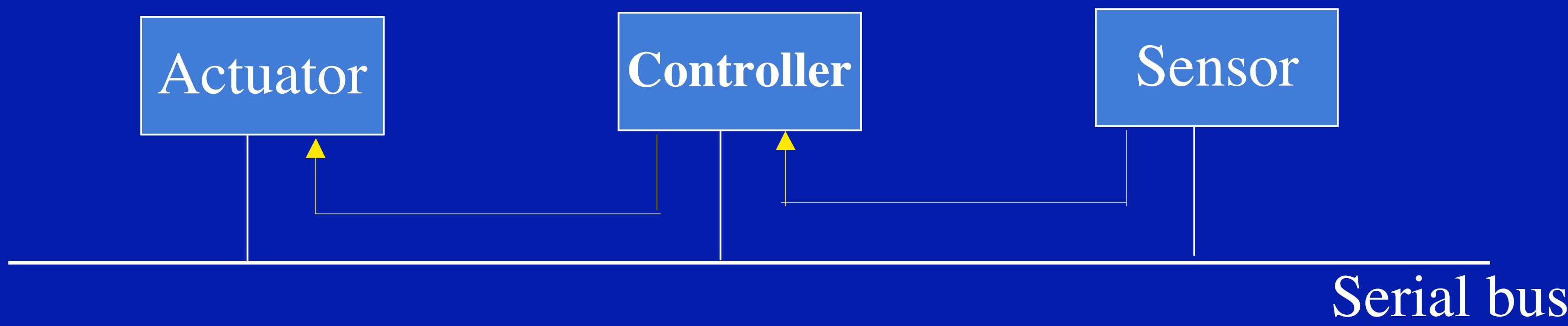
$y_k = h_k(x_k) + v_k$ $M < N$

$u_k = \mu_k(I_k)$, $I_k := \{y_{[0,k]}, u_{[0,k-1]}\}$, $\mu_k \rightarrow U_k$ only M times

Given a horizon of N units, and with controller allowed to transmit for only M < N times, what is the minimum attainable value of a performance index J, and a corresponding controller?

PROBLEM CLASS III

Scheduled Measurements & Controls



$$x_{k+1} = A\, x_k + a_k\, u_k + w_k\ ,\quad k = 0,\ldots, N-1$$

a_k is 0 or 1

$$y_k = (1 - a_k)\, x_k\ ,\ k = 0,\ldots, N-1 \quad \text{measurement}$$

Controller cannot receive and send simultaneously

Solutions

In all cases, solutions are of the **threshold** type.

For example, for Problem Class I, best sensor policy is of the form:

At time k transmit z_k if it is in a measurable set ¥(s_k,t_k), otherwise do not.

¥(s,t) obtained offline as the minimizer in a recursive equation satisfied by accumulated optimum error, $e^*(s,t)$, at each point (s,t), where t is # decision instances left, and s is # usages left.

Specific structure of ¥(s,t) depends on the pdf/pmf and PI.

Smart Sensing

Input parameters:

1. Planning horizon: Desired length of time the sensor will be in operation (N)
2. Battery size: Size of the power source installed into the sensor (M)
3. Process model (data): This is application specific, and can be gathered from a variety of sources including, measurement/historic data, modeling, etc. (Source Statistics)
4. Performance criterion: (Distortion Measure)

Smart Sensing

How?

- Given the input parameters and the channel compute the optimum transmission and decoding policies for the wireless sensor & the receiver
- The computation is "power aware" (M), and "planning-horizon aware" (N)
- Minimal online computational complexity (lookup table, binary comparisons,…)
- Offline computational complexity will depend on the source and channel models

To conclude …

- Networks & Control
- Issues in Networked Control Systems

  -- What/when/how to transmit & control
- Control with Unreliable Channels
- Control with Power-Limited Communication

  -- When to measure & when to control
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Networked Control Systems is a current, active, and fertile research topic offering interesting challenges in both theory and applications.

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Thanks !

- Conclusions