CDMA Uplink Power Control as a Noncooperative Game [1]

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28th September 2001

Abstract

We present a game-theoretic treatment of distributed power control in CDMA wireless systems. We make use of the conceptual framework of noncooperative game theory to obtain a distributed and market-based control mechanism. Thus, we address not only the power control problem, but also pricing and allocation of a single resource among several users. A cost function is introduced as the difference between the pricing and utility functions, and the existence of a unique Nash equilibrium is established. In addition, two update algorithms, namely parallel update and random update, are shown to be globally stable under specific conditions. Convergence properties and robustness of each algorithm are also studied through extensive simulations.

Keywords: Power control, CDMA, game theory, pricing, resource allocation

[1]Research supported in part by grants NSF ANI 98-13710, NSF INT 98-04950, NSF CCR 00-85917, AFOSR MURI AF DC 5-36128, and ARMY OSP 35352-6086. To be presented at the 2001 IEEE Conference on Decision and Control, Orlando, Florida, December 4-7, 2001, and a shorter version to appear in the Conference Proceedings. All correspondence should be forwarded to: Prof. Tamer Başar, Coordinated Science Laboratory, University of Illinois, 1308 West Main Street, Urbana, IL 61801-2307 USA. Tel: (217) 333-3607; Fax: (217) 244-1653; E-mail: tbasar@control.csl.uiuc.edu

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1 Introduction

In wireless communication systems, mobile users respond to the time-varying nature of the channel, described using short-term and long-term fading phenomena [1], by regulating their transmitter powers. Specifically, in a code division multiple access (CDMA) system, where signals of other users can be modeled as interfering noise signals, the major goal of this regulation is to achieve a certain signal to interference (SIR) ratio. Hence, there are two major reasons for a user to exercise power control: the first one is the limit on the battery energy available to the mobile, and the second reason is the increase in capacity, which can be achieved by minimizing the interference.

Power control in CDMA systems are in either open-loop or closed-loop form. In open-loop power control, the mobile regulates its transmitted power inversely proportional to the received power. In closed-loop power control, on the other hand, commands are transmitted to the mobile over the downlink to increase or decrease its uplink power [2, pp.182].

A specific proposal to implement distributed power control made by Yates [3] relies on each user updating its power based on the total received power at the base station. It has been shown in [3] that the resulting distributed power control algorithm converges under a wide variety of interference models. Another distributed power control scheme has been introduced in [4], which is adaptive and uses local measurements of the mean and the variance of the interference. The authors have shown that this algorithm is convergent provided that a certain condition is satisfied.

Game theory provides a natural framework for developing pricing mechanisms of direct relevance to the power control problem in wireless networks. In such networks, the users behave noncooperatively, i.e., each user attempts to minimize its own cost function (or maximize its utility function) in response to the actions of the other users. This makes the use of noncooperative game theory [5] for uplink power control most appropriate, with the relevant solution concept being the noncooperative Nash equilibrium. In this approach, a noncooperative network game is defined where each user attempts to minimize a specific cost function by adjusting his transmission power, with the remaining users' power levels fixed. The main advantage of this approach is that it not only leads to distributed control as in [3], but also naturally suggests pricing schemes, as we will see in this paper.

Possible utility functions in a game theoretical framework, and their properties for both voice and data sources have been investigated in detail in [6], which formulates a class of utility functions that also account for error correction, and shows the existence of a unique Nash equilibrium. One interesting feature of this framework is that it provides utility functions for wireless data transmission, where power control directly affects the capacity of mobiles' data transmission rates. Reference [6] also proposes a linear pricing scheme in order to achieve a Pareto improvement in the utilities of mobiles.

In an earlier study [7], Nash equilibria achieved under a pricing scheme have been characterized by using supermodularity. It has been shown that a noncooperative power control game with a pricing scheme is superior to one without pricing. One deficiency of this game setup, however, is that it does not guarantee social optimality for the equilibrium points.

In the model we adopt in this paper, we use a cost function defined as the difference between a linear pricing scheme proportional to transmitted power, and a logarithmic, strictly concave utility function based on SIR of the mobile. We then prove the existence and uniqueness of a Nash equilibrium. As in [3], one way of extending the model is to include certain SIR constraints. As an alternative, we suggest a pricing strategy to meet the given constraints, and analyze the relation between price and SIR. We study different pricing strategies, and obtain a sufficient condition for convergence of two algorithms, parallel update (PUA) and random update (RUA), to the unique Nash equilibrium.

In order to illustrate the convergence, stability and robustness of the update algorithms, we use extensive simulations using MATLAB. Moreover, we study the effect of the various parameters of the model, especially different pricing schemes. In order for the simulations to capture realistic scenarios, we introduce feedback delay and modeling disturbances, where the latter is caused by variations in the number of users in the network.

The next section describes the model adopted and the cost function. In Section 3, we prove the existence and uniqueness of the Nash equilibrium. We present update algorithms for mobiles in Section 4, whereas Section 5 introduces different pricing strategies at the base station. The simulation results are given in Section 6, which is followed by the concluding remarks of Section 7.

2 The Model and Cost Function

We describe here the simple model adopted in this paper for a single cell CDMA system with up to M users. The number of users is limited under an admission control scheme that ensures the minimum necessary SIR for each user in the cell. For the i^{th} user, we define the cost function J_i as the difference between the utility function of the user and its pricing function, $J_i = P_i - U_i$. The utility function, U_i, is chosen as a logarithmic function of the i^{th} user's SIR, which we denote by γ_i. This utility function can be interpreted as being proportional to the Shannon capacity [1] for user i, if we make the simplifying assumption that the noise plus the interference of all other users constitute an independent Gaussian noise. This means that this part of the utility is simply linear in the throughput that can be achieved (or approached) by user i using an appropriate coding, as a function of its transmission power. This logarithmic function is further weighted by a user-specific utility parameter, $u_i > 0$, to capture the user's level of "desire" for SIR.

The pricing function defines the instantaneous "price" a user pays for using a specific amount of power that causes interference in the system. It is a linear function of p_i, the power level of the user. Accordingly, the cost function of the i^{th} user is defined as

$$J_i(p_i, p_{-i}) = \lambda_i \, p_i - u_i \ln(1 + \gamma_i) \;, \quad p_i > 0 \; \forall i, \tag{2.1}$$

where p_{-i} denotes the vector of power levels of all users except the i^{th} one, and γ_i is the SIR function for user i, given by

$$\gamma_i = L\frac{h_i p_i}{\sum_{j\neq i} h_j p_j + \sigma^2} \ . \tag{2.2}$$

Here, $L = \frac{W}{R}$ is the spreading gain of the CDMA system, where W is the chip rate and R is the total rate; we assume throughout that $L > 1$. The parameter h_j, $0 < h_j < 1$, is the channel gain from user j to the base station in the cell, and $\sigma^2 > 0$ is the interference. For notational convenience, let us denote the i^{th} user's power level received at the base station as $y_i := h_i p_i$, introduce the quantity $\bar{y}_{-i} := \sum_{j\neq i} y_j$, and further define a user specific parameter (a_i) for the i^{th} user as

$$a_i := \frac{u_i h_i}{\lambda_i} - \frac{\sigma^2}{L} \ . \tag{2.3}$$

3 Existence and Uniqueness of Nash Equilibrium

The i^{th} user's optimization problem is to minimize its cost, given the sum of powers of other users as received at the base station, $\bar{y}_{-i}$, and noise. The nonnegativity of the power vector $(p_i \geq 0 \ \ , \forall i)$ is an inherent physical constraint of the model. Taking the derivative of the cost function (2.1) with respect to p_i, we obtain the first-order necessary condition:

$$\frac{\partial J_i(p_i, p_{-i})}{\partial p_i} = \lambda_i - \frac{L u_i h_i}{\sum_{j\neq i} h_j p_j + L h_i p_i + \sigma^2} \geq 0 \tag{3.1}$$

In the case of a positive inner solution, (3.1) holds with equality. It is easy to see that the second derivative is also positive, and hence the inner solution, if it exists, is the unique point minimizing the cost function. The boundary solution, $p_i = 0$, is the other possible optimal point for the constrained optimization problem. If the user's cost function, $J_i(p_i, p_{-i})$, attains its minimum for a power value less than zero, $p_{i,min} < 0$, the optimal solution will be the boundary point. Solving Equation (3.1) and invoking the positivity constraint $p_i \geq 0$, we obtain the reaction function, Φ_i, of the i^{th} user:

$$p_i = \Phi_i(\bar{y}_{-i}, a_i) = \begin{cases} \frac{1}{h_i}[a_i - \frac{1}{L}\bar{y}_{-i}], & \text{if } \bar{y}_{-i} \leq La_i \\ 0 & , \text{ else} \end{cases} \tag{3.2}$$

The reaction function is the optimal response of the user to the varying parameters in the model. It depends not only on the user-specific parameters, like u_i, λ_i, and h_i, but also on the network parameter, L, and total power level received at the base station, $\sum_{j=1}^{M} y_j$. Actually (3.2) shows dependence only on $\bar{y}_{-i}$, but adding $(-\frac{1}{L}p_i)$ to both sides, and dividing both sides by $(1 - \frac{1}{L})$, one can express the response of the i^{th} user as a function of the quantity $\sum_{j=1}^{M} y_j$. Similar to the transmission control protocol (TCP) in the Internet [8], there is an inherent feedback mechanism here, built into the reaction function of the user. In this model, the total received power at the base station provides the user with information about the "demand" in the network, which is comparable to congestion in case of the TCP. However, one major difference is that here the reaction function itself takes the place of the window based algorithms in the TCP.

We observe the following conditions from (3.2) in order for the mobile to be "active," or $p_i > 0$. The first condition is the positivity of $a_i > 0$ for the i^{th} user to be active and it follows from the fact that both channel gains and power levels of all users must be positive, $h_j, p_j > 0 \ \ , \forall j$. Using the definition of a_i this can be equivalently interpreted as a lower bound on the utility parameter u_i. If $a_i \leq 0$, by the positivity of channel gains and power levels of users it follows that :

$$\bar{y}_{-i} \geq 0 \geq La_i \ , \tag{3.3}$$

and from (3.2) $p_i = 0$. Otherwise, we obtain a second condition

$$\bar{y}_{-i} \leq La_i \ , \tag{3.4}$$

as given in (3.2). Both positivity of a_i and (3.4) need to be satisfied in order for the mobile to be active. An intuitive interpretation for these conditions is the following: If the price, λ_i, is set too high for a mobile, the mobile prefers not to transmit at all, depending on his channel gain and utility parameter, and the spreading gain and interference level.

In reality, positivity of power is only a necessary, and not a sufficient condition for a mobile to establish communication with the base station. On top of that, we may also need to set lower bounds on the SIRs of the users.

The relationship between SIR, number of users, and price will be further investigated in this paper for different pricing schemes. For any equilibrium solution, the set of fixed point equations can be written in matrix form by exploiting the linearity of (3.2). In case of a boundary solution, the rows and columns corresponding to users with zero equilibrium power are deleted, and the equation below involves only the users with positive powers. Hence we have (assuming here that all M users have positive power levels at equilibrium):

$$\begin{pmatrix} 1 & \frac{h_2}{Lh_1} & \frac{h_3}{Lh_1} & \cdots & \frac{h_M}{Lh_1} \\ \frac{h_1}{Lh_2} & 1 & \frac{h_3}{Lh_2} & \cdots & \frac{h_M}{Lh_2} \\ \frac{h_1}{Lh_3} & \frac{h_2}{Lh_3} & 1 & \cdots & \frac{h_M}{Lh_3} \\ \vdots & \vdots & & \ddots & \vdots \\ \frac{h_1}{Lh_M} & \frac{h_2}{Lh_M} & \cdots & \frac{h_{M-1}}{Lh_M} & 1 \end{pmatrix} \begin{pmatrix} p_1^* \\ \vdots \\ p_i^* \\ \vdots \\ p_M^* \end{pmatrix} = \begin{pmatrix} c_1 \\ \vdots \\ c_i \\ \vdots \\ c_M \end{pmatrix} \Leftrightarrow \mathcal{A}\mathbf{p}^* = \mathbf{c} \quad , \tag{3.5}$$

where the variable c_i is defined as $c_i = \frac{a_i}{h_i}$. Note that $c_i > 0$ if a_i is positive.

Theorem 3.1. *In the power game just defined (with M users), let the indexing be done such that $a_i < a_j \Rightarrow i > j$, with the ordering picked arbitrarily if $a_i = a_j$. Let $M^* \leq M$ be the largest integer $\tilde{M}$ for which the* *following condition is satisfied:*

$$a_{\tilde{M}} > \frac{1}{(L+\tilde{M}-1)} \sum_{i=1}^{\tilde{M}} a_i \ . \tag{3.6}$$

Then, the power game admits a unique Nash equilibrium (NE), which has the property that users $M^*+1,\ldots,M$ have zero power levels, $p_j^* = 0 \ j \geq M^*+1$. The equilibrium power levels of the first M^* users are obtained uniquely from (3.5) with M replaced by M^*, and are given by

$$p_i^* = \frac{1}{h_i}\{\frac{L}{L-1}[a_i - \frac{1}{L+M^*-1}\sum_{j\in\mathbf{M}^*} a_j]\}, \ i \in \mathbf{M}^* := \{1,2,\ldots,M^*\} \ . \tag{3.7}$$

If there is no $\tilde{M}$ for which (3.6) is satisfied, then the NE solution is again unique, but assigns zero power level to all M users.

Proof. We first state and prove the following lemma, which will be useful in the proof of the theorem.

Lemma 3.2. *If condition (3.6) is satisfied for $\tilde{M} = \hat{M}$, it is also satisfied for all $\tilde{M}$ such that $1 \leq \tilde{M} < \hat{M}$.*

Proof. Suppose that condition (3.6) holds for $\tilde{M} = \hat{M}$; then we argue that it also holds for $\tilde{M} = \hat{M}-1$. Substituting in (3.6) $\hat{M}$ for $\tilde{M}$, we rewrite it as:

$$(L+\hat{M}-2)a_{\hat{M}} - a_{\hat{M}-1} > \sum_{i=1}^{\hat{M}-2} a_i \ .$$

Due to the indexing of users, we have $a_{\hat{M}-1} \geq a_{\hat{M}}$. Substituting $a_{\hat{M}-1}$ for $a_{\hat{M}}$ above, we obtain

$$(L+\hat{M}-3)a_{\hat{M}-1} > \sum_{i=1}^{\hat{M}-2} a_i \ .$$

Hence, (3.6) is satisfied for $\tilde{M} = \hat{M}-1$. The proof then follows by induction on $\tilde{M}$. □

Returning to the proof of the theorem, we first show that the matrix $\mathcal{A}$ in equation (3.5) is full rank and hence invertible, and thereby the solution to (3.5), $\mathbf{p}^*$, is unique. Then we show that the solution is strictly positive if, and only if, condition (3.6) is satisfied for $\tilde{M} = M$. Finally, we relax condition and allow for boundary solutions, and conclude the proof by proving the uniqueness of the boundary solution.

In order for the matrix $\mathcal{A}$ in (3.5) to be full rank and hence invertible, there should not exist a nonzero vector $q = (q_1 \; q_2 \dots q_M)^T \neq 0$ such that $\mathcal{A}q = 0$. This equation can be written as the following set of equations:

$$Lh_i q_i + \sum_{j \neq i} h_j q_j = 0 \quad \forall i$$

$$\Rightarrow (L-1)h_i q_i + \sum_{j=1}^{M} h_j q_j = 0 \quad \forall i \tag{3.8}$$

Summing up this set of equations over all users, $i = 1, \dots, M$, we arrive at:

$$(L-1+M)(\sum_{j=1}^{M} h_j q_j) = 0$$

It is clear that the term $L-1+M$ above is nonzero, and hence the sum $\sum_{j=1}^{M} h_j q_j$ has to be zero. Since the channel gains are strictly positive, $h_i > 0 \; \forall i$, and $L > 1$, it follows from (3.8) that $q_i = 0 \;\; \forall i$. Accordingly, the matrix $\mathcal{A}$ is full rank and hence invertible, which leads to a unique solution for equation (3.5). Simple manipulations lead to the expression (3.7), with $M^* = M$, for this unique solution.

If the NE exists and is strictly positive, then (3.5) has to have a unique positive solution, which we already know is given by (3.7). Hence, (3.7) has to be positive, which is precisely condition (3.6) in view of also the indexing of the users. On the other hand, if (3.6) holds for $\tilde{M} = M$, then we obtain from (3.7) that the equilibrium power level of each user is strictly positive. The existence and uniqueness of the NE follows from (3.5). We thus conclude that condition (3.6) with $\tilde{M} = M$ is both necessary and sufficient for the existence of a unique positive Nash equilibrium.

To complete the proof for the case $M^* = M$, possible boundary solutions need to be investigated to conclude the uniqueness of the inner Nash equilibrium. We have to show that there cannot be another NE, with a subset $\tilde{\mathbf{M}}$ of $\tilde{M}$ users transmitting with positive power, and the remaining $M - \tilde{M}$ users having zero power level. In this case, the reactive power level of the i^{th} mobile, $i \in \tilde{\mathbf{M}}$, is given by (3.7) with $M^* = \tilde{M}$.

For any i^{th} mobile, $i \notin \tilde{\mathbf{M}}$, in order for the zero power level to be part of a Nash equilibrium, condition

$$\bar{y}_{-i} \leq L a_i \tag{3.9}$$

should fail according to the reaction function (3.2) of the mobile. Summing up the equilibrium power levels as received by the base station of $\tilde{M}$ users with positive power levels (from (3.7) with $M^* = \tilde{M}$) results in

$$\frac{1}{L} \sum_{j \in \tilde{\mathbf{M}}} y_j = \frac{1}{L + \tilde{M} - 1} \sum_{j \in \tilde{\mathbf{M}}} a_j \tag{3.10}$$

Substituting in (3.9) the expression (3.10) for $\bar{y}_{-i}$ yields

$$a_i \geq \frac{1}{L + \tilde{M} - 1} \sum_{j \in \tilde{\mathbf{M}}} a_j \ . \tag{3.11}$$

On the other hand, from Lemma 3.2, and (3.6), we have for any i^{th} user in the indexed set $\{1, \ldots, \tilde{M} + 1\}$ the following

$$a_i \geq \frac{1}{L + \tilde{M} - 1} \sum_{j=1}^{\tilde{M}} a_j \ . \tag{3.12}$$

Also, from the indexing of the users it follows that $\sum_{j=1}^{\tilde{M}} a_j \geq \sum_{j \in \tilde{\mathbf{M}}} a_j$. Using this in (3.12), we see that inequality (3.11) is satisfied for any i^{th} user $i \in \{1, \ldots, \tilde{M} + 1\}$ regardless of the choice of the subset $\tilde{\mathbf{M}}$. We note that there exists at least one user belonging to the set $\{1, \ldots, \tilde{M} + 1\}$, but not the subset $\tilde{\mathbf{M}}$. Thus, the power of that mobile must be positive, and hence the boundary solution cannot be a Nash equilibrium. As this argument is valid for any subset $\tilde{\mathbf{M}}$, all boundary solutions fail similarly for being an equilibrium, including the trivial solution, the origin. We thus conclude that the inner Nash equilibrium is unique. This completes the proof for the case $M^* = M$.

If $M^* < M$ in condition (3.6), then the equilibrium (whenever it exists) will clearly be a boundary point. If condition (3.6) fails for users $M^* + 1, \ldots, M$ where users are indexed as described in Theorem 3.1, then these users use zero power in the equilibrium. Hence, for any i^{th} user among $M^*+1,\ldots,M$, condition (3.9) should fail. It was shown above that equation (3.10) holds with $\tilde{M}=M^*$. As condition (3.6) does not hold for the i^{th} user, equation (3.11), and hence (3.9) fails. Thus, from (3.2) power level of the i^{th} user is zero, $p_i=0$, at the equilibrium. As this holds for any $i \in \{\hat{M}+1,\ldots,M\}$, the equilibrium power levels for these users are zero.

We now argue that the given boundary solution is unique. One possibility is the existence of an i^{th} user, where $1 \leq i \leq M^*$, to have zero power. This cannot be a Nash equilibrium, as it follows from (3.10) and (3.11) with $\tilde{M}=M^*$. Another possibility is the existence of an i^{th} user, where $M^* \leq i \leq M$, transmitting with positive power level. This cannot be an equilibrium, either, as it was shown above that (3.9) fails for such an i^{th} user, and $p_i=0$ follows directly from the reaction function (3.2). All possible boundary solutions can be captured by various combinations of these two cases. Consider the case where a subset of users among $1,\ldots,M^*$ use zero power whereas some of the users among $M^*+1,\ldots,M$ use positive power levels. Since for the subset of users with positive power levels among $M^*+1,\ldots,M$ condition (3.6) does not hold, they cannot be in equilibrium following an argument similar to the one above. Otherwise, as condition (3.6) holds for the subset of users with zero power level among $1,\ldots,M^*$, they cannot be in equilibrium, either. We conclude, therefore, that the boundary solution is unique.

Finally, in the case where no M^* exists satisfying condition (3.6), all users fail to satisfy (3.6), and the only solution is the trivial one, $p_i^*=0 \; \forall i$. □

4 Update Schemes for Mobiles, and Stability

In this section, we investigate the stability of the Nash equilibrium in the given model under two relevant asynchronous update schemes: parallel and random update. We establish a sufficient condition which guarantees the convergence to the unique equilibrium point for both algorithms.

4.1 Parallel Update Algorithm (PUA)

In the PUA, users optimize their power levels at each iteration (in discrete time intervals) using the reaction function (3.2). If the time intervals are chosen to be longer than twice the maximum delay in the transmission of power level information, it is possible to model the system as an ideal, delay-free one. In a system with delays, there are subsets of users, updating their power levels given the delayed information.

One important feature of PUA is that it ascribes a myopic behavior to the users, that is they optimize their power levels based on instant costs and parameters, ignoring future implications of their actions. In a delay-free system, this behavior affects convergence rate adversely as it will be seen in the simulations.

The algorithm is given by

$$p_i^{(n+1)} = \Phi_i(\bar{y}_{-i}^{(n)}, a_i) = \max(0, \frac{1}{h_i}[a_i - \frac{1}{L}\sum_{j \neq i} y_j^{(n)}]) \ , \ \ y_j^{(n)} = h_j p_j^{(n)} \ , \tag{4.1}$$

or equivalently by

$$y_i^{(n+1)} = \max(0, a_i - \frac{1}{L}\sum_{j \neq i} y_j^{(n)}) \ ,$$

whose global stability is established in the next theorem. This means that PUA converges to the unique Nash equilibrium of Theorem 3.1 given as

$$p_i^* = \max(0, \frac{1}{h_i}[a_i - \frac{1}{L}\sum_{j \neq i} h_j p_j^*]) \tag{4.2}$$

from any feasible initial point, $p_i \geq 0 \ \forall i$.

Theorem 4.1. *PUA is globally stable, and converges to the unique equilibrium solution from any feasible starting point if the following condition is satisfied:*

$$\frac{M-1}{L} < 1 \ . \tag{4.3}$$

Proof. Let us define the distance between the i^{th} user's power level received in the base station at any time (n) and received equilibrium power level as $\triangle y_i^{(n)} := y_i^{(n)} - y_i^*$. We consider first the case when $y_i^* > 0$ for an arbitrary i^{th} user. Then, given the received power levels of all users except the i^{th} one at time n, $y_j^{(n)}$ $j \neq i$, we have the following from (4.1) and (4.2):

$$\triangle y_i^{(n+1)} = \begin{cases} -a_i + (\bar{y}^*_{-i}/L) \ , & \text{if} \quad a_i < (\bar{y}_{-i}^{(n)}/L) \\ \frac{1}{L} \sum_{j \neq i} \triangle y_j^{(n)} \ , & \text{else} \end{cases} ,$$

from which it follows that

$$|\triangle y_i^{(n+1)}| \begin{cases} < \frac{1}{L} | \sum_{j \neq i} \triangle y_j^{(n)} | \ , & \text{if} \quad a_i < (\bar{y}_{-i}^{(n)}/L) \\ = \frac{1}{L} | \sum_{j \neq i} \triangle y_j^{(n)} | \ , & \text{else} \end{cases}$$

Thus, we obtain

$$|\triangle y_i^{(n+1)}| \leq \frac{1}{L} \sum_{j \neq i} |\triangle y_j^{(n)}| \tag{4.4}$$

Next, we consider the case when the received equilibrium power level for an arbitrary i^{th} user is zero, $y_i^* = 0$. Then, from (4.1) and (4.2) it follows that

$$|\triangle y_i^{(n+1)}| \leq \begin{cases} \frac{1}{L} | \sum_{j \neq i} \triangle y_j^{(n)} | \ , & \text{if} \ a_i > (\bar{y}_{-i}^{(n)}/L) \\ 0 \ , & \text{else} \end{cases}$$

Thus, the inequality (4.4) holds for any i^{th} user at any time instant n for both cases. We now show that (4.3) is a sufficient condition for the right-hand side of (4.4) to be a contraction mapping. Let $||\triangle y||_\infty$ denote the l_∞-norm of the vector $(\triangle y_1 \ \triangle y_2 \ \ldots \triangle y_M)^T$, i.e.,

$$||\triangle y||_\infty = \max_i |\triangle y_i| \tag{4.5}$$

Then, from (4.4),

$$||\triangle y^{(n+1)}||_\infty \leq \frac{1}{L} \max_i \sum_{j \neq i} |\triangle y_j^{(n)}| \leq \frac{M-1}{L} ||\triangle y^{(n)}||_\infty \ .$$

Hence, (4.4) is a contraction mapping under condition (4.3), which leads to the stability and global convergence of the PUA (4.1). □

We finally note that using an initial admission control mechanism and user dropping scheme, which limits the number, M, of users in the cell, this condition can easily be satisfied for a given L. Thus, the stability and convergence of the algorithm follows.

4.2 Random Update Algorithm (RUA)

Random update scheme is a stochastic modification of PUA. The users optimize their power levels in discrete time intervals and infinitely often, with a predefined probability $0 < \pi_i < 1$. Thus, at each iteration a set of randomly picked users among the M update their power levels. Again, the users are myopic and make instantaneous optimization. In the limiting case, $\pi_i = 1$, RUA is the same as PUA. The non-ideal system with delay is also similar to PUA. The users make decisions based on delayed information at the updates, if the round trip delay is longer than the discrete time interval.

The RUA algorithm is described by

$$p_i^{(n+1)} = \begin{cases} \Phi_i(\bar{y}_{-i}^{(n)}, a_i), & \text{with probability } \pi_i \\ p_i^{(n)}, & \text{with probability } 1 - \pi_i \end{cases},$$

or equivalently by

$$y_i^{(n+1)} = \begin{cases} h_i \Phi_i(\bar{y}_{-i}^{(n)}, a_i), & \text{with probability } \pi_i \\ y_i^{(n)}, & \text{with probability } 1 - \pi_i \end{cases},$$

where Φ_i was defined in (4.1). We already know from the proof of Theorem 4.1 that if user i updates, then (4.4) holds. Hence, for each $i = 1, \ldots, M$,

$$
\begin{aligned}
E|\triangle y_i^{(n+1)}| &= E\{|\triangle y_i^{(n+1)}| \,\Big|\text{user i updates at time n}\}\pi_i \\
&\quad + E\{|\triangle y_i^{(n)}| \,\Big|\text{user i does not update at time n}\}(1-\pi_i) \\
&\leq \frac{\pi_i}{L}\sum_{j\neq i} E|\triangle y_j^{(n)}| + (1-\pi_i)E|\triangle y_i^{(n)}| \qquad (4.6)
\end{aligned}
$$

Using again the l_∞-norm defined in (4.5), but with $|\triangle y_i|$ replaced by $E|\triangle y_i|$ (that is, $||\triangle y||_\infty := \max_i E|\triangle y_i|$), and following steps similar to the ones of PUA, we obtain

$$
\max_i E|\triangle y_i^{(n+1)}| \leq \frac{M-1}{L}||\triangle y^{(n)}||_\infty \max_i \pi_i + \max_i (1-\pi_i)||\triangle y^{(n)}||_\infty
$$

which leads to

$$
||\triangle y^{(n+1)}||_\infty \leq (\frac{M-1}{L}\bar{\pi} + (1-\underline{\pi}))||\triangle y^{(n)}||_\infty \ ,
$$

where $\bar{\pi} < 1$ and $\underline{\pi} > 0$ are the upper and lower limits for the update probability of the i^{th} user respectively, $\underline{\pi} < \pi_i < \bar{\pi}$. Therefore,

$$
\frac{M-1}{L}\bar{\pi} + (1-\underline{\pi}) < 1 \qquad (4.7)
$$

is a sufficient condition for the right-hand side of (4.6) to be a contraction mapping, and for the stability and convergence of RUA in norm. We also note that when all users have the same update probability, $\pi_i = \pi \ \forall i$, this condition simplifies to $(M-1)/L < 1$, same sufficient condition (4.3) as the one for PUA. We show next a stronger result, almost sure (a.s.) convergence of RUA, under condition (4.7). By the Markov inequality and using the definition of the l_∞-norm, we have

$$
\sum_{n=1}^{\infty} P(|\triangle y_i^{(n)}| > \varepsilon) \leq \sum_{n=1}^{\infty} \frac{E|\triangle y_i^{(n)}|}{\varepsilon} \leq \frac{1}{\varepsilon}\sum_{n=1}^{\infty} ||\triangle y^{(n)}||_\infty \ , \qquad (4.8)
$$

where $P(.)$ denotes the underlying probability measure. Since $E|\triangle y_i^{(n)}|$ is a contracting sequence with respect to the l_∞-norm as shown,

$$||\triangle y^{(n)}||_\infty \leq \alpha ||\triangle y^{(n-1)}||_\infty \leq \cdots \leq \alpha^n ||\triangle y^{(0)}||_\infty \ ,$$

where $0 < \alpha < 1$. Using this in (4.8), it follows that

$$\sum_{n=1}^{\infty} P(|\triangle y_i^{(n)}| > \varepsilon) \leq \sum_{n=1}^{\infty} \alpha^n \frac{||\triangle y^{(0)}||_\infty}{\varepsilon} = \frac{||\triangle y^{(0)}||_\infty}{\varepsilon(1-\alpha)} \ ,$$

and thus follows,

$$\sum_{n=1}^{\infty} P(|\triangle y_i^{(n)}| > \varepsilon) \leq \frac{K}{\varepsilon(1-\alpha)} \ ,$$

where K is a constant (actually, $K = ||\triangle y^{(0)}||_\infty$). Hence, the increasing sequence of partial sums $\sum_{n=1}^{N} P(|\triangle y_i^{(n)}| > \varepsilon)$ is bounded above by $\frac{K}{\varepsilon(1-\alpha)}$. Thus, it converges for every $\varepsilon > 0$. From the Borel-Cantelli Lemma [9, 10], it then follows that

$$P(\limsup_{n\to\infty} \{\omega : |\triangle y_i^{(n)}| > \varepsilon\}) = 0 \ .$$

Hence, RUA converges also a.s. under condition (4.7).

5 Pricing Strategies at the Base Station

In a noncooperative network, pricing is an important design tool as it creates an incentive for the users to adjust their strategies, in this case power levels, in line with the goals of the network. In the CDMA system we are studying here, the price per unit power of the i^{th} user, λ_i, is determined by the base station in a manner to be discussed shortly. We introduce a pricing scheme where the price charged to each user is proportional to the received power from the user at the base station. Thus, the price is proportional to the channel gain of the i^{th} user, $\lambda_i = k_i h_i$.

The inner Nash solution by itself does not guarantee that the users with nonzero power levels will meet the minimum SIR requirement to establish a connection to the base station. Achieving the necessary SIR level is obviously crucial to the successful operation of the system. Furthermore, one has to recognize that different communication applications in wireless systems leads to different types of users and SIR requirements in addition to the minimum SIR level.

In view of these considerations, we will consider in this section two different pricing schemes:

(i) A *centralized pricing scheme:* Users are divided into classes, with all users belonging to a particular class having the same utility function parameter (u_i). Further, all users within a class have the same SIR requirement. The role of the base station is to set prices for these different classes such that, under the resulting Nash equilibrium, the SIR targets of the users are met.

(ii) *Decentralized, market-based pricing:* The base station sets a single price for all users, and the users choose their willingness to pay parameter, (u_i), to satisfy their QoS requirements. As compared to the centralized scheme, this one is more flexible, and allows users to compete for the system resources by adjusting their individual u_i's.

5.1 Centralized Pricing Schemes and Admission Control

First, we consider the symmetric-user case where every mobile has the same SIR requirement, and for convenience we let $u_i = 1$. It is possible to find a simple pricing strategy by picking the price directly proportional to the channel gain, $\lambda_i = kh_i$, where the pricing factor, k, is user independent. The parameter k is a function of the number of users and the desired SIR level.

Notice that this approach is equivalent to centralized power control as the prices are adjusted by the base station in such a way that the mobiles use the power levels determined by the unique Nash equilibrium as a result of their individual optimization. Moreover, the base station can set the prices such that the SIR requirements of the users are satisfied. A precise result covering this case is now captured by the following theorem.

Theorem 5.1. *Assume that the users are symmetric in their utilities, $u_i = 1\ \forall i$, they have the same minimum SIR requirement, γ^*, and are charged proportional to their channel gain, $\lambda_i = kh_i$. Then the maximum number of users, M^*, the system can accomodate is bounded by*

$$M^* < \frac{L}{\gamma^*} + 1 \ . \tag{5.1}$$

Moreover, the pricing parameter k under which $M \le M^*$ users achieve the SIR level γ^* is

$$k = \frac{\lambda_i}{h_i} = \frac{L}{\sigma^2} \frac{L - \gamma^*(M-1)}{L(\gamma^*+1)} \ . \tag{5.2}$$

Proof. Solving for the user-independent y_i from (3.2), we have

$$y_i = \frac{(L/k) - \sigma^2}{L + M - 1}$$

Combining this result and the SIR function in (2.2), and taking the minimum SIR, γ^*, as input, we obtain (5.2) for a single class of users in a cell. To ensure that (5.2) is well defined, we require the condition in (5.1). Based on (5.2), condition (5.1) satisfies (3.6). Thus, both the necessary and sufficient conditions for a unique positive Nash equilibrium are satisfied if (5.1) holds. Then, the unique solution is strictly positive according to Theorem 3.1, and all $M \le M^*$ users attain the desired SIR level, γ^*. □

We note that if $M > M^*$, all users fall below the desired SIR level (γ^*) due to the symmetry. In this case, dropping some of the users from the system in order to decrease the number of users M below the threshold (M^*) would lead to a viable solution.

Next, we consider the case where the network may provide multiple service levels and multiple pricing schemes. For this more general case, it is convenient to split the mobiles in a cell into multiple groups according to their need for bandwidth, or in our context, their desired SIR levels, where the users within each group are symmetric. Using a multiple pricing scheme, a solution capturing multiple user groups can be obtained. It is again possible to choose this scheme to be socially fair, in the sense that mobiles are provided the same service level within a specific group regardless of their distance to the base station.

We will investigate for simplicity the two-group case with symmetric users within each group such that the users of the same group have the same SIR requirement. It is straightforward to extend this two group pricing scheme to higher number of user groups with different SIR requirements. However, the market-based scheme (discussed in the next subsection) provides a more suitable solution for such cases.

In the multiple pricing scheme, we define the prices $\lambda_i^{(1)} = k^{(1)} h_i$ and $\lambda_j^{(2)} = k^{(2)} h_j$, for groups 1 and 2, respectively. Suppose that group 1 is for multimedia applications, hence requiring a high SIR. Decreasing $k^{(1)}$ yields the desired result for such users. Note that, prices in this scheme should be considered in terms of network credits instead of actual price a user pays. Because of the interaction between users of different groups, the function (5.2) cannot be used to determine the value of the pricing parameters $k^{(1)}$ and $k^{(2)}$. In order to find the appropriate values, we make use of the symmetry of users within a group and derive a relation similar to (5.2).

Let the target SIR levels for the two groups with $N^{(1)}$ and $N^{(2)}$ users be $\gamma^{(1)}$ and $\gamma^{(2)}$, respectively. Furthermore, let u_i be written as $u^{(1)}$ for the first and $u^{(2)}$ for the second group. Rewriting (2.2),

$$\gamma^{(1)} = \frac{L y_i}{N^{(2)} y_j + (N^{(1)} - 1) y_i + \sigma^2}$$
$$\gamma^{(2)} = \frac{L y_j}{(N^{(2)} - 1) y_j + N^{(1)} y_i + \sigma^2},$$
$$i = 1, \ldots, N^{(1)} \; ; \; j = 1, \ldots, N^{(2)} \; ,$$

and solving for y_i and y_j in this set of equations, we obtain:

$$y_i = \frac{\gamma^{(1)}\sigma^2(\gamma^{(2)}+L)}{-\gamma^{(1)}\gamma^{(2)}N^{(1)}N^{(2)} + [\gamma^{(1)}(N^{(1)}-1)-L][\gamma^{(2)}(N^{(2)}-1)-L]} \quad (5.3)$$

$$y_j = \frac{\gamma^{(2)}\sigma^2(\gamma^{(1)}+L)}{-\gamma^{(1)}\gamma^{(2)}N^{(1)}N^{(2)} + [\gamma^{(1)}(N^{(1)}-1)-L][\gamma^{(2)}(N^{(2)}-1)-L]} \quad (5.4)$$

Notice that the received power level, and hence SIR, is the same for all users within each group.

Finally, we establish a relation between the pricing factors $k^{(1)}$ and $k^{(2)}$ and the power levels within each group as received by the base station, y_i and y_j:

$$k^{(1)} = \frac{u^{(1)}L}{(L+N^{(1)}-1)y_i + (N^{(2)})y_j + \sigma^2} \quad (5.5)$$

$$k^{(2)} = \frac{u^{(2)}L}{(L+N^{(2)}-1)y_j + (N^{(1)})y_i + \sigma^2} \quad (5.6)$$

Combining (5.3) - (5.6) provides the pricing strategy for the two-group case in terms of the desired SIR levels, number of users, system parameter L, and noise level σ^2. Following the lines of the derivation above, the set of linear equations above can be easily extended to three or higher classes, leading to a general pricing strategy.

The expressions in (5.3 , 5.4) are of course required to be nonnegative. If they are negative for some mobiles, then it is not possible to achieve the desired SIR level for the given parameters. In this case, the Nash equilibrium will be a unique boundary solution according to the second part of Theorem 3.1. By implementing an appropriate dropping scheme, users can be dropped beginning with the one having lowest channel gain to price ratio, a_i, until the remaining users meet condition (3.6). Thus, a unique inner Nash equilibrium exists for the game with active users, in view of Theorem 3.1. The solution for all the mobiles, including the inactive ones, is on the other hand a unique boundary solution in accordance with Theorem 3.1.

5.2 Market-Based Scheme

It is natural to think of each user within a cell having different SIR requirements, which can be quantified with the user-specific utility parameter u. The base station can implement a natural pricing strategy by formulating the pricing parameter directly proportional to the channel gain, $\lambda_i = kh_i$. However, it is impossible in this case for the base station to calculate the parameter k, as the user preferences are unknown to the base station. Hence, after the base station sets an appropriate value for price (k), each user dynamically updates its power level by minimizing its cost under parallel update (PUA) or random update (RUA) algorithms. As a result, a distributed and market-based power control scheme is obtained.

Due to the interference in the CDMA system, each user affects others. Hence, the i^{th} mobile can adjust its utility parameter, u_i, dynamically according to its minimum SIR level, γ_i^*, given the interference at the base station. From (3.2) and (2.2), it follows that

$$u_i > \frac{\lambda_i}{Lh_i}(\gamma_i^* + 1)(\bar{y}_{-i} + \sigma^2)$$

The parameter u_i is bounded below by the total received power at the base station. This can be interpreted as follows: If a mobile is in a cell where the interference is low, the mobile can achieve the desired SIR level with a low power, hence paying a lower price. However, in a situation where many users compete for the SIR, the mobile has to use more power, and pay a higher price to reach the same SIR level. In the latter case, the user's willingness to spend more can be justified with a higher u_i based on (3.2).

We note that, together with the utility function, the utility parameter u_i quantifies the user's desire for the SIR. The base station can limit aggressive requests for SIR even in the case when a user pays for its excessive usage of power, by setting an upper-limit, y_{max}, for the received power of the i^{th} user at the base station: $y_i \leq y_{max}$. Hence, unresponsive users can be punished by the base station in order to preserve network resources. From (3.2), we can obtain an upper-bound on the value of u_i. Furthermore, this bound depends only on user-independent parameters, such as the upper limit of the total received power at the based station, maximum number of mobiles, M_{max}, and the spreading gain, L, if proportional pricing is used:

$$u_i \leq \frac{k}{L}[\sigma^2 + (L + M_{max} - 1)y_{max}] \ \ ,\forall i.$$

When this bound is combined with a simple admission control scheme, limiting the number of mobiles to M_{max}, the base station can provide guarantees for a minimum SIR level γ_{min}:

$$\gamma_{min} = \frac{L y_{max}}{(M_{max} - 1)y_{max} + \sigma^2}$$

A tradeoff is observed in the choice of the design parameters γ_{min} versus M_{max}. If the network wants to provide guarantees for a high SIR level, then it has to make a sacrifice by limiting the number of users. In addition, users may implement a distributed admission scheme according to their budget constraints and desired SIR levels. If the necessary price to reach a SIR level exceeds the budget, B_i, of the user, that is $\lambda_i(\gamma_i^* + \bar{y}_{-i} + \sigma^2)/L u_i h_i \geq B_i$, then the user may simply choose not to transmit at all.

6 Simulation Studies

The proposed power control scheme has been simulated numerically using MATLAB. Here, we first investigate different pricing schemes for both symmetric and multiple groups of mobiles in the fixed-utility case. Then, we analyze the robustness of the system under varying parameters such as noise, the number of users, and channel gains. Finally, we investigate a system consisting of users with various utility parameters. All results of the simulations are valid for both update schemes, PUA and RUA, where the only difference between the two is the convergence rate.

Simulation parameters are chosen as follows, unless otherwise stated: spreading gain $L = 800$, noise $\sigma^2 = 10$, the stopping criterion or distance to equilibrium $\epsilon = 10^{-5}$. The channel gains of users are determined randomly with uniform distribution, $0.2 < h_i < 1$, to be more realistic. Under the fixed-utility case, users have the same utility parameter, $u_i = 1$,$\forall i$. The initial condition for simulations is $p_i = 1$,$\forall i$, an estimated value for establishing initial communication between the mobile and the base station. In the simulations, a discrete time scale is used. The delay-free case is characterized by a time span that is long enough for perfect information flow to users. Subsequently, delay is introduced to the system to make the setting more realistic.

6.1 Effect of the Pricing Parameters

In the first simulation, proportional and fixed pricing schemes are compared. For simplicity, we first choose the users being symmetric under both fixed pricing, $\lambda_i = \lambda$, and proportional pricing, $\lambda_i = kh_i$. For illustrative purposes the number of users is chosen small, $M = 20$.

In Figure 1, the equilibrium power and the SIR values of each user can be seen under both pricing schemes. In the top graph, power values of the users with different channel gains are almost the same under fixed pricing. Hence, the users with lower channel gains fail to meet the minimum SIR goal, chosen arbitrarily as 10 in the middle graph. In contrast, all users meet the minimum SIR level under proportional pricing, regardless of their channel gain. An intuitive explanation is that under proportional pricing the distant users are allowed to use more power to attain the necessary SIR. We also note that, proportional pricing is 'fair' in the sense that the users are not affected by their distance to the base station.

Convergence of users' power levels to their equilibrium values is demonstrated in Figure 2 (a) under PUA, and Figure 2 (b) under RUA with update probability being 0.6. In both cases there are 10 users and $L = 20$.

The effect of pricing is investigated in the next simulation for a single class of users by varying the pricing parameter, k, under proportional pricing. Equivalently, this simulation can be interpreted as varying the

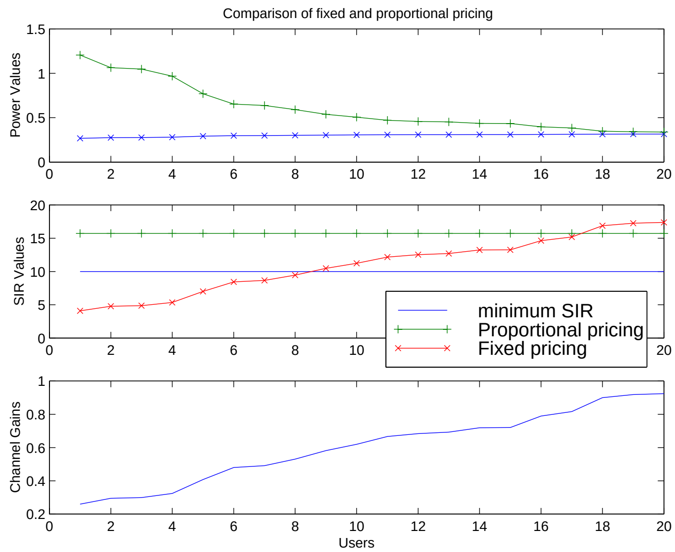
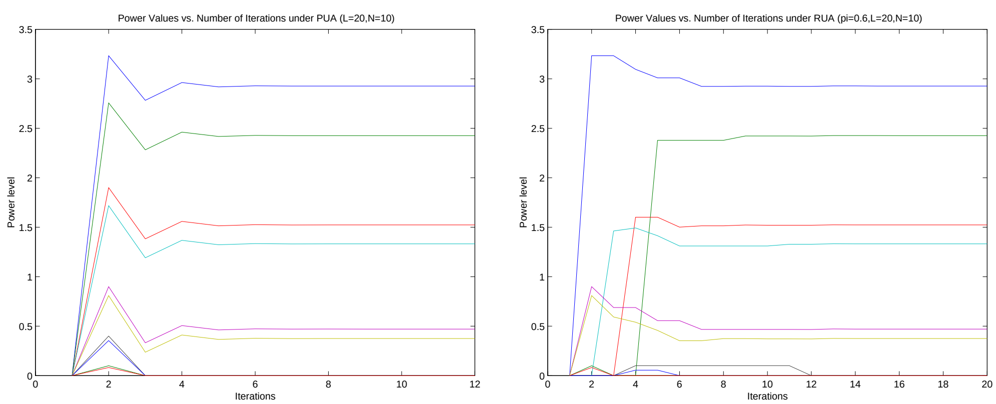


Figure 1: Comparison of power and SIR final values of the mobiles for the fixed and proportional pricing schemes.



(a) (b)

Figure 2: Convergence of users' individual power levels to the equilibrium values versus number of iterations under (a) PUA, and (b) RUA with $\pi = 0.6$.

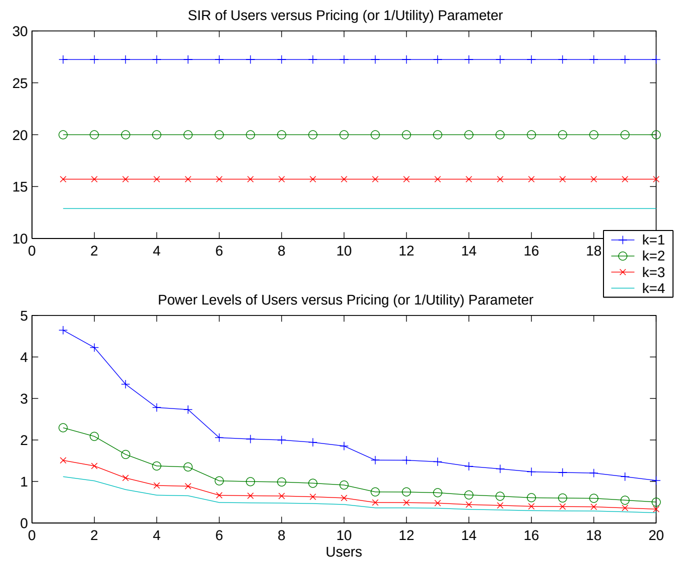
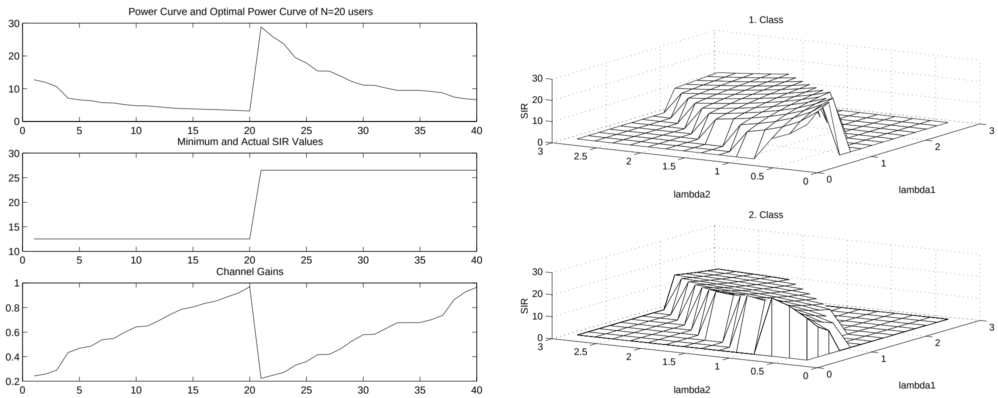


Figure 3: Effect of the pricing parameter, k, (utility parameter $1/u$) on the SIR and the power levels of users.

utility parameter, u. Both parameters play a crucial role in the system by affecting the overall power and SIR levels. From (3.2), the effect of u_i on the system is inversely proportional to k_i. In Figure 3 it can be observed that a gradual increase in k from 1 to 4, i.e. an increase in price, affects the system in a such a way that both power and SIR values decrease. Since, with an increase in the price, the users decrease their powers to the same extent leading to lower SIR values given a constant noise level. Equivalently, a decrease in u, the users' level of request for SIR, gives the same result. Furthermore, the observations match theoretical calculations for the single class case in accordance with (5.2).

As an example of multiple pricing strategies within a single cell, we have chosen the two group case for illustrative purposes. It is assumed that users have a fixed-utility, $u_i = 1\ ,\forall i$. One group is given priority against the other one in terms of its SIR level. Similar to Figure 1, Figure 4 (a) shows the SIR and the power values of 40 users, with 20 in each class under proportional pricing. The user group with lower prices achieves a higher SIR, as expected. The results of this simulation justify the previous calculations in Section 5.



(a) (b)

Figure 4: Two groups of users: (a) SIR, power, channel gains, and (b) SIR levels vs. pricing parameters.

Figure 4 (b) summarizes the SIR results for each group of mobiles under varying proportional prices: $k^{(1)}$ and $k^{(2)}$. The minimum SIR level is again chosen as 10. Parameters are zeroed out when they cannot provide this required SIR. As expected, the difference between parameters $k^{(1)}$ and $k^{(2)}$ is roughly proportional to the difference between SIR levels of the two classes. The diagonal line represents equal pricing, hence the symmetric case.

6.2 Convergence Rate and Robustness of Algorithms

6.2.1 Simulations without Delay

The convergence rate of the two update schemes is of great importance, as it has a direct effect on the robustness of the system. We have simulated PUA and RUA for different numbers of symmetric users under a single pricing scheme. In Figure 5, the number of iterations to the equilibrium point is shown for different probability values of RUA and also for PUA (which corresponds to RUA with the update probability equal to one). As the number of users increase the optimal update probability decreases. This result is in accordance with the one in [11] where it is shown that in a quadratic system without delay, an approximate value for the optimal update probability is $\frac{2}{3}$, as number of users goes to infinity. On the other hand, if number of users is much smaller than the spreading gain, $M \ll L$, then PUA is superior to RUA.

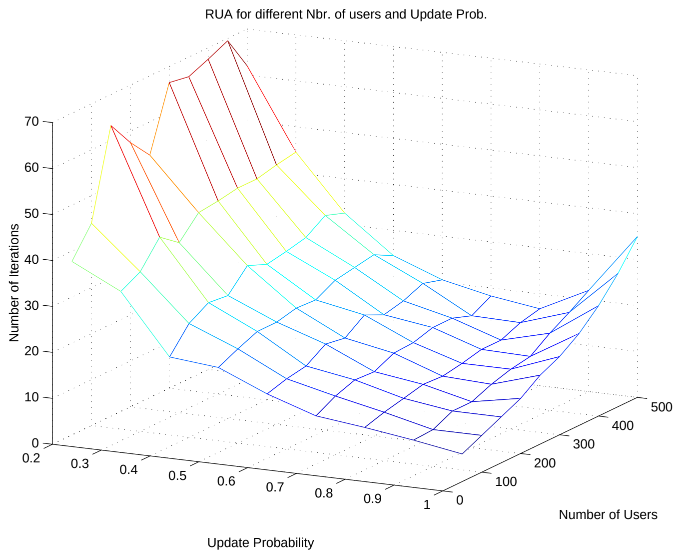
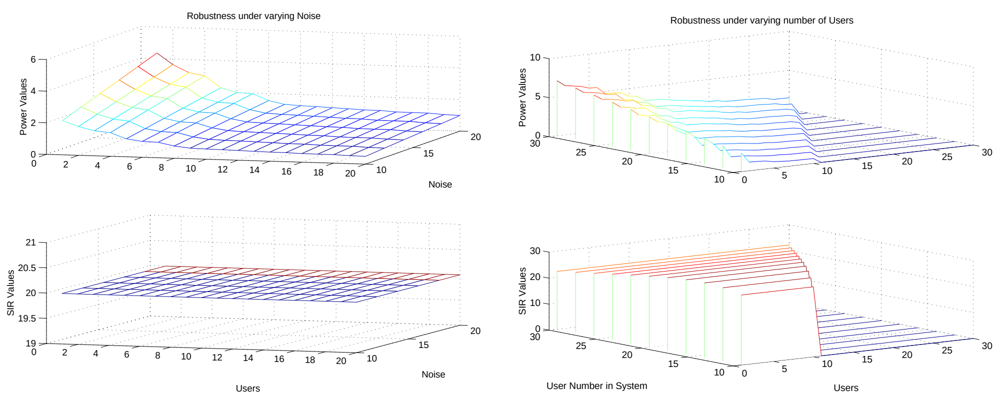


Figure 5: Convergence rate for different update probabilities and increasing numbers of users.

Next, we investigate the robustness of the system in the ideal, delay-free case. First, we analyze it under increasing noise, σ^2. The background noise is increased step by step up to 100% of its initial value. Accordingly, the base station allows users to increase their powers by decreasing the prices by the same percentage in the fixed-utility case. The simulation is repeated with $N = 20$ users under a proportional pricing scheme. We observe in Figure 6 (a) that the power values increase in response to the increasing noise to keep the initial SIR constant. Similarly, we increase the number of mobiles in the system threefold in Figure 6 (b). It has the same effect as increasing the noise due to the nature of CDMA. Again by adjusting the prices accordingly, all users keep their SIR levels. Same results are obtained equivalently under the market-based pricing scheme, where users adjust their utility parameter, u, dynamically while the pricing parameter determined by the base station is kept constant. As a conclusion, these observations confirm the robustness of the proposed power control scheme.



(a) (b)

Figure 6: Power and SIR final values for increasing noise (a) and numbers of users (b).

Finally, we simulate the system in a realistic setting under a single pricing scheme: The number of users, $N = 10$ taken as initial value, is modeled as a Markov chain. Arrival of new mobiles is chosen to be Poisson with an average of 2 new users per time interval. Call durations are exponentially distributed with an average of 20 time intervals. We observe the average percentage difference between the theoretical equilibrium and the current operating point of the system in terms of power values of users for some period of time. In the simulation, PUA is chosen as the update algorithm. The initial condition is the equilibrium point for users. In Figure 7, it is shown that the system operates within 1% range of ideal equilibrium points.

Heretofore, robustness of the system was investigated for static mobiles. The movements of the users within the cell can be modeled by changing the channel gains randomly with time. In the next simulation, the channel gains of users are varied randomly up to 15% of their previous values at each time instant. Furthermore the setting used in this simulation is the same as previous one. From Figure 8, the system again operates within 1% distance to ideal equilibrium.

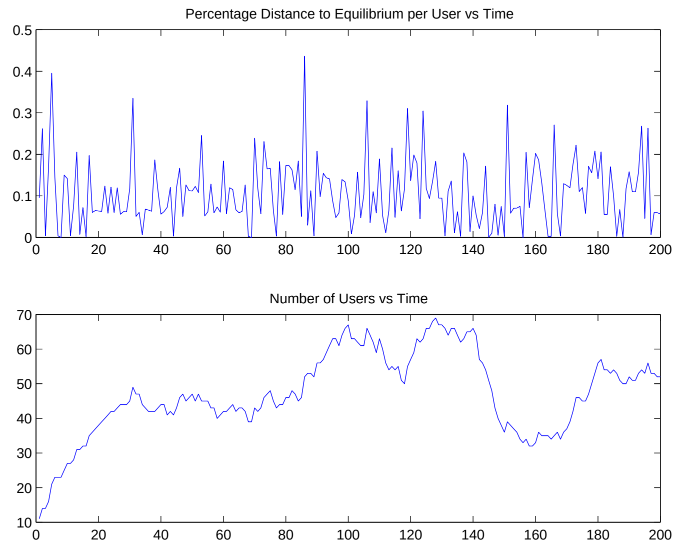


Figure 7: Average percentage distance to equilibrium point vs. time. The number of users is modeled as a Markov chain.

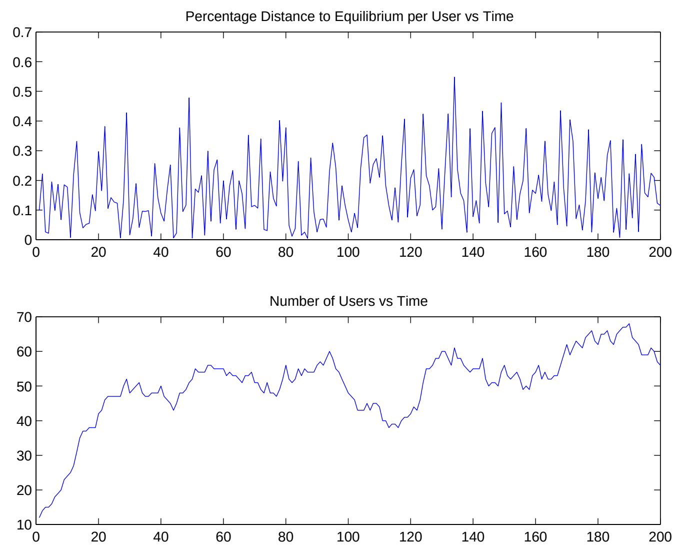


Figure 8: Average percentage distance to the equilibrium point vs. time. Channel gains, h_i, are varied up to 15% per unit time.

6.2.2 Simulations with Delay

We introduce the delay factor into the system in the following way: users are divided into d equal size groups, and each group has an increasing number of units of delay.

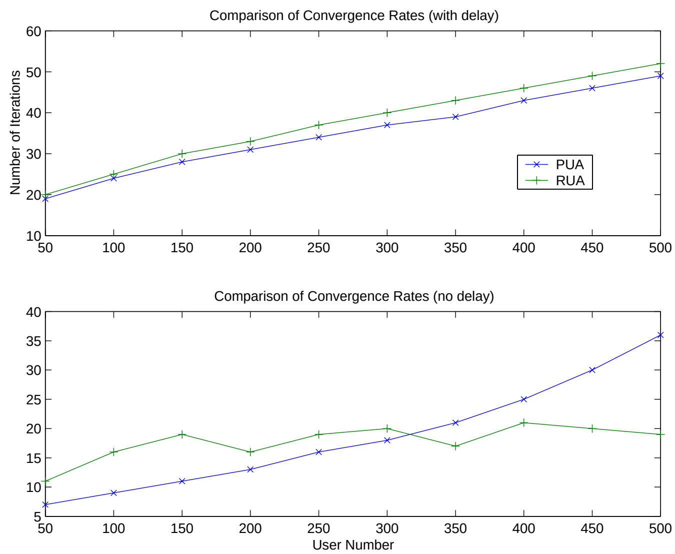


Figure 9: Comparison of the convergence rates of PUA and RUA for increasing numbers of users in the no delay (bottom graph) and with delay (top graph) cases.

First, the convergence rates of the two update schemes are compared and contrasted under ideal and delayed conditions. The update probability of RUA is chosen as 0.66 which corresponds to the optimal update probability for a large number of users. It can be seen in Figure 9 that in the ideal case RUA outperforms PUA as the number of users increases. In the delayed case, however, PUA is always superior to RUA.

Then, the simulation investigating the convergence rate of RUA for various update probabilities is repeated in the delayed case. The result shown in Figure 10 is different from the previous one in Figure 5. Here, PUA converges faster than RUA for any number of users verifying the results in Figure 9.

Finally, a market-based pricing scheme with proportional pricing at the base station, $k = 1$, is investi-

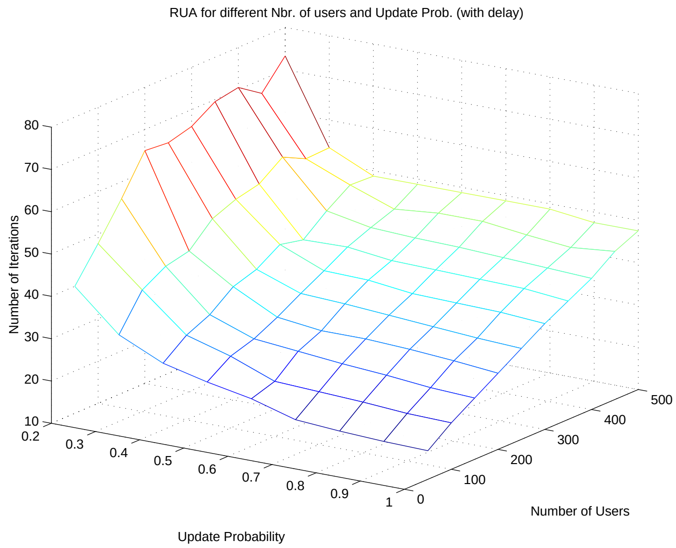


Figure 10: Convergence rate for different update probabilities and increasing numbers of users (with delay)

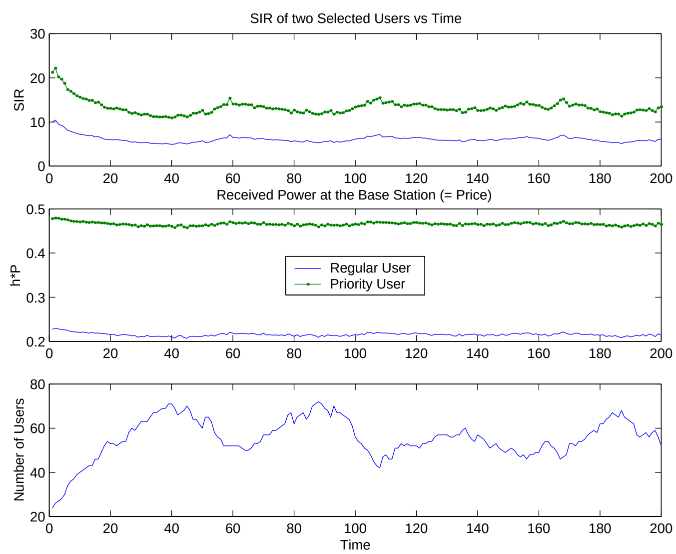


Figure 11: SIR and power levels at the base station (or prices) of two selected users from priority and regular user groups versus time.

gated. There are two groups of users, which are symmetric within themselves. Users in each group have different utility parameters, u. The group with higher u is labeled as the "priority" user group, while the other one is called the "regular" user group. In order to observe the effect of varying number of users on the SIR levels, we let a sample user from each group make a long enough call. At the same time, the number of users in each group and channel gains of the users are varied similar to those in previous robustness simulations to create realistic disturbances in the system. For simplicity, the values of the utility parameters are kept constant throughout the simulation. In Figure 11, it is observed that a priority user always obtains a higher SIR than a regular user. The fluctuation of the SIR of both users suggests that users should update their utility parameters dynamically to compensate for the changes in the network. Another observation is that priority users use a higher power level, and therefore pay more than regular users, as expected. The fluctuation in the prices is due to the varying number of users, and varying total demand for SIR in the system.

7 Conclusion

In this paper, we have developed a mathematical model within the framework of noncooperative game theory, and have obtained distributed, asynchronous control mechanisms for the uplink power control problem in a single cell CDMA wireless network. Existence of a unique Nash equilibrium has been proven, and convergence properties of parallel and random update schemes have been investigated analytically and numerically. Moreover, conditions for the stability of the unique equilibrium point under the update algorithms have been obtained and analyzed accordingly.

We have shown that the unique Nash equilibrium has the property that, depending on the parameter values, only a subset of the total number of mobiles are active. Some of the users are dropped from the system as a result of the power optimization. By defining a utility function and a utility parameter, user requests for SIR were modeled dynamically. Furthermore, the relationship between the SIR level of the users and the pricing has been investigated for two different pricing schemes for the fixed and varying utility cases. It has been shown both analytically and through simulations that choosing an appropriate pricing strategy guarantees meeting the minimum desired SIR levels for the active users in the fixed-utility case. In addition, the principles for an admission scheme have been investigated under the market-based scheme.

The results obtained indicate that the game theoretical approach can provide satisfactory decentralized and market-based solutions. There still exist, however, some open questions, which require further investigation. One possible extension to this work is to a multiple cells model, where the effect of neighboring cells are taken into account. Another topic of research is the development if the counterparts of the results in the case of multiple base stations, which brings up the challenging issue of handoff.

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